BLIND NONZERO DELAY MMSE EQUALIZER FOR SIMO FIR SYSTEMS

Abla Kammoun[1], *Karim Abed-Meraim*[1] *and Sofiène Affès*[2]

[1] Télécom ParisTech France, [2] INRS-EMT Canada

ABSTRACT

Recently, a novel method for blind channel equalization based on the truncation of the covariance matrix has been proposed. Despite having interesting properties, more specifically its low complexity through adaptive implementation and its robustness to channel over-modeling, this method, is based on zero delay equalization thus yielding non satisfactory results in case the first channel coefficient is of low power. In this paper, we propose to generalize this method to nonzero delay equalization. We show that the proposed method not only inherits the same interesting properties of the original one, but also improves considerably its performance and its sensitivity to the value of the first channel coefficient.

Index Terms— MMSE equalization, blind equalization, nonzero delay equalization

1. INTRODUCTION

Blind channel methods can be classified into two categories. One is the indirect blind approach for which a priori estimation of the channel is required. This approach usually requires singular value decomposition of the output correlation matrix, thus making real-time implementation difficult [1]. The other approach namely, the direct blind method, estimates the optimum linear equalizer by using the second order statistics of the data without involving any estimation of the channel filter. This method can be implemented by using adaptive algorithms that exhibit low-cost computational complexity.

Recently, active research work has been made in order to develop efficient techniques for direct blind equalization methods, with low computational complexity and that are robust to channel over-modeling. In [2], a mutually referenced filter based method has been proposed. This technique requires the estimation of several (at least two) equalizers, and thus is not computationally efficient.

On the other hand, various methods involving the estimation of a unique equalizer, have been widely proposed. For instance, one can cite linear prediction based techniques that were proposed to estimate nonzero delay equalizers, [1, 3]. These methods are slightly robust to channel order over-modeling errors [4], but their adaptive implementations are either expensive (e.g., RLS like) or slowly convergent (e.g., LMS like).

Later, new methods based on the truncation of the covariance matrix have been proposed [5, 6]. These techniques yield zero delay equalizers through performing an appropriate truncation to the covariance matrix. Despite their high robustness to channel over-modeling, these methods do not always yield satisfactory results, since they involve zero delay equalization and thus are sensitive to the value of the first channel coefficient.

In this paper, we propose to generalize the methods in [5, 6] to nonzero delay equalization. We show by using simulations that our method allows significant performance improvement while maintaining robustness to channel order over-modeling.

The organization of this paper is as follows: First we describe the system model in section II. Then, in the next section we review and state some results about the method proposed in [5]. Based on these results, we introduce our method in section IV. Finally, we provide and discuss the simulation results.

Notations Throughout this paper, vectors and matrices are respectively represented by boldface small and capital letters. Moreover, the transpose, hermitian, complex conjugate inverse and expectation operators are denoted by $^{\mathrm{T}}$, $^{\mathrm{H}}$, *, $^{-1}$ and $\mathbb{E}$. We also denote by $\|\mathbf{x}\|$ the Euclidean norm of vector $\mathbf{x}$. We adopt some MATLAB notations like $A(k,:)$ or $A(:,k)$ to refer to the kth row or kth column of matrix A.

2. SYSTEM MODEL

Consider a discrete-time Single Input Multiple Output (SIMO) system with M outputs, given by:

$$\mathbf{x}(n) = \sum_{k=0}^{L} \mathbf{h}(k)s(n-k) + \mathbf{b}(n),$$

where $s(k)$ denotes the transmitted symbol sequence and $\mathbf{h}(k)$ refers to the $M \times 1$ channel impulse response vector corresponding to the kth tap. $\mathbf{b}(n)$ denotes the white noise sequences with variance σ_b^2. Stacking N successive observations of the received signal $\mathbf{x}(n)$ into a single vector, we get:

$$\mathbf{x}_N(n) = [\mathbf{x}^{\mathrm{T}}(n), \cdots, \mathbf{x}^{\mathrm{T}}(n-N+1)]^{\mathrm{T}} \quad (1)$$

$$= \mathbf{H}_N \mathbf{s}_d(n) + \mathbf{b}_N(n), \quad (2)$$

where $d = N + L$, $\mathbf{s}_d(n) = [s(n), \cdots, s(n-d+1)]^{\mathrm{T}}$ and $\mathbf{b}_N(n) = [\mathbf{b}^{\mathrm{T}}(n), \cdots, \mathbf{b}^{\mathrm{T}}(n-N+1)]^{\mathrm{T}}$. The matrix $\mathbf{H}_N$ is the $NM \times d$ block toeplitz matrix given by:

$$\mathbf{H}_N = \begin{bmatrix} \mathbf{h}(0) & \cdots & \mathbf{h}(L) & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \mathbf{h}(0) & \cdots & \mathbf{h}(L) \end{bmatrix}.$$

In the sequel, we consider the following additive assumptions:

A1 The transmitted signal $s(n)$ is assumed to be an independent and identically distributed zero mean unit power process.

A2 The polynomial $\mathbf{h}(z) = \sum_{k=0}^{L} \mathbf{h}(k) z^{-k}$ verifies:

$$\mathbf{h}(z) \neq 0 \ \text{ for all } \ z \neq 0,$$

or equivalently, the sub-channels corresponding to the receiving antennas do not share any zero in common.

3. MMSE EQUALIZER

We recall hereafter the equalization technique that has been proposed in [5]. The main features of this technique are its highest robustness to channel over-modeling and also its low computational complexity as compared to other proposed techniques.

As we will see below, this method is based on the fact that the τ-delay MMSE equalizer belongs to a certain vector space that depends solely on the autocorrelation matrix, and whose dimension is equal to $\tau + 1$. As a consequence, if $\tau = 0$, the zero-delay MMSE equalizer can be estimated up to a scalar ambiguity.

For the reader's convenience, we provide in the sequel an overview on the main results derived in [5]. Let us first recall that the τ-delay linear MMSE equalizer ($\tau \in \{0, \cdots, d-1\}$) is the optimal linear filter that extracts $s(n-\tau)$ in the least square sense.

More explicitly, the linear MMSE equalizer vector $\mathbf{v}_\tau$ is given by:

$$\mathbf{v}_\tau = \arg\min_{\mathbf{v}} \mathbb{E}\left(\| s(n-\tau) - \mathbf{v}^{\mathrm{H}} \mathbf{x}_N(n) \|^2 \right),$$

which leads to:

$$\mathbf{v}_\tau = \mathbf{R}^{-1} \mathbf{g}_\tau, \tag{3}$$

where

$$\mathbf{R} = \mathbb{E}\left(\mathbf{x}_N(n) \mathbf{x}_N^{\mathrm{H}}(n)\right) = \mathbf{H}_N \mathbf{H}_N^{\mathrm{H}} + \sigma_b^2 \mathbf{I}_{MN}$$

and

$$\mathbf{g}_\tau = \mathbb{E}\left(\mathbf{x}_N(n) s^*(t-\tau)\right) = \mathbf{H}_N(:, \tau+1).$$

One can note that the linear MMSE equalizer belongs to the signal subspace, i.e., Range($\mathbf{H}_N$) and thus can be written as:

$$\mathbf{v}_\tau = \mathbf{W} \tilde{\mathbf{v}}_\tau,$$

where $\mathbf{W}$ denotes the signal subspace basis vectors. Along the same lines as in [5], we can prove the following result:

Theorem 1. *Let $\underline{\mathbf{R}}_\tau$ be the matrix given by the last $MN - (\tau+1)M$ rows of $\mathbf{R}$. Then, assuming* **A1**, **A2**, *and that $N > L + 1 + \tau$, the kernel of matrix $\mathbf{W}^{\mathrm{H}} \underline{\mathbf{R}}_\tau^{\mathrm{H}} \underline{\mathbf{R}}_\tau \mathbf{W}$ has dimension $\tau + 1$ and contains all the t-delay equalizers $\mathbf{v}_t$, $t \in \{0, \cdots, \tau\}$.*

The proof of this theorem has not been provided before, but it relies on the same technique used in [5]. For the sake of completeness, we provide hereafter the proof of this theorem:

Proof. Let $\mathbf{R} = \mathbf{W}\mathbf{\Gamma}\mathbf{W}^{\mathrm{H}} + \sigma_b^2 \mathbf{U}\mathbf{U}^{\mathrm{H}}$ be the eigenvalue decomposition of $\mathbf{R}$, where $\mathbf{W}$ and $\mathbf{U}$ are the eigenvectors that span, respectively, the signal and the noise subspace. Since, the columns of $\mathbf{W}$ and $\mathbf{U}$ are orthogonal, we have:

$$\mathbf{R}\mathbf{W} = \mathbf{W}\mathbf{\Gamma}.$$

Hence, Range($\mathbf{R}\mathbf{W}$) = Range($\mathbf{W}$) = Range($\mathbf{H}_N$). Therefore, there exists a nonsingular matrix $\mathbf{P}$ such that $\mathbf{R}\mathbf{W} = \mathbf{H}_N \mathbf{P}$.

As a consequence,

$$\underline{\mathbf{R}}_\tau \mathbf{W} = \left[0_{(N-\tau-1)M \times \tau+1} \ \mathbf{H}_{N-\tau-1} \right] \mathbf{P}.$$

We end up the proof by noting that if $N - \tau - 1 > L$, $\mathbf{H}_{N-\tau-1}$ is full column rank [7] and thus $\dim(\mathrm{null}(\underline{\mathbf{R}}_\tau \mathbf{W})) = \tau + 1$. □

Example: Zero Delay Equalizer

Let $\mathbf{v}_0$ denote the zero delay equalizer. There exists a vector $\tilde{\mathbf{v}}_0$ such that $\mathbf{v}_0 = \mathbf{W}\tilde{\mathbf{v}}_0$, with

$$\mathbf{R}\mathbf{W}\tilde{\mathbf{v}}_0 = \mathbf{g}_0 = \begin{bmatrix} \mathbf{h}(0) \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}.$$

Let $\underline{\mathbf{R}}_0$ be the matrix given by the last $MN - M$ rows of $\mathbf{R}$. Then the following result has been shown in [5] and can be easily deduced from Theorem 1.

Corollary 1. *Assuming that $N > L + 1$, the solution of*

$$\underline{\mathbf{R}}_0 \mathbf{W} \tilde{\mathbf{v}} = \mathbf{0},$$

is unique and corresponds to the desired zero-delay equalizer vector $\mathbf{v}_0$ up to a scalar ambiguity.

In other words, this result states that the zero delay equalizer can be determined as the intersection line between the range space of $\mathbf{W}$ and the kernel of the matrix $\underline{\mathbf{R}}_0$.

In the nonzero delay case, this intersection becomes a vector space of dimension $\tau + 1$, that contains all the t delay equalizers, $t \in \{0, \cdots, \tau\}$. The issue now is how to select in this vector space the direction of the desired τ delay equalizer. In [5], a two-step approach is proposed: First, the zero delay equalizer is estimated and the transmitted symbols are decoded by performing a hard decision on the equalized signal. After that, the estimated symbols are reused to reestimate a nonzero delay equalizer according to (3).

However, one may expect that this technique will not provide good performance, since as soon as the result of the first step is bad, all the process that comes after, will be affected.

4. NON-ZERO DELAY EQUALIZER

The performance of the zero delay equalizer is poor when the energy of the first channel coefficient $\mathbf{h}(0)$ is low. In average, the nonzero delay equalizer has better performance since it depends on more than one channel tap and thus can benefit from the channel path diversity. Motivated by this well known result [8], we propose in this paper to extend the work of [5] to the nonzero delay case.

Let $\mathcal{F}_\tau$ denote the vector space given by the intersection between the kernel of $\underline{\mathbf{R}}_\tau$ and the range of $\mathbf{W}$, and $\mathbf{B}_\tau = \{\tilde{\mathbf{v}}_0, \cdots, \tilde{\mathbf{v}}_\tau\}$ a basis of $\mathcal{F}_\tau$. Then, obviously, we have $\mathcal{F}_{\tau-1} \subset \mathcal{F}_\tau$. More particularly, a basis of $\mathcal{F}_\tau$ could be given as the union of $\mathbf{B}_{\tau-1}$ and another vector $\tilde{\mathbf{v}}_\tau$ which cannot be written as a linear sum of elements of $\mathbf{B}_{\tau-1}$.

The difficulty here is to select the right direction $\tilde{\mathbf{v}}_\tau$, which corresponds to the τ-delay equalizer. To solve this problem, we will make use of an approximate orthogonality relation between the equalizer vectors, which is accurate as long as the mean square symbol estimation error is low. Indeed, since $\mathbf{v}_t^{\mathrm{H}}\mathbf{x}_N(n) \simeq s(n-t)$ and $\mathbf{v}_{t'}^{\mathrm{H}}\mathbf{x}_N(n) \simeq s(n-t')$, and since the input symbols are i.i.d., we get:

$$0 = \mathbb{E}\left(s(n-t)s^*(n-t')\right) \simeq \mathbf{v}_t^{\mathrm{H}}\mathbf{R}\mathbf{v}_{t'}. \quad (4)$$

More explicitly, if $\mathbf{R} = \mathbf{W}\mathbf{\Gamma}\mathbf{W}^{\mathrm{H}} + \sigma_b^2\mathbf{U}\mathbf{U}^{\mathrm{H}}$ denotes the eigenvalue decomposition of $\mathbf{R}$, then, since $\mathbf{v}_t$ belongs to the range space of $\mathbf{W}$, (4) is equivalent to:

$$\mathbf{v}_t^{\mathrm{H}}\mathbf{W}\mathbf{\Gamma}\mathbf{W}^{\mathrm{H}}\mathbf{v}_{t'} \simeq 0.$$

Let $\check{\mathbf{v}}_\tau = \mathbf{\Gamma}^{\frac{1}{2}}\mathbf{W}^{\mathrm{H}}\mathbf{v}_\tau$, i.e, $\mathbf{v}_\tau = \mathbf{W}\mathbf{\Gamma}^{\frac{-1}{2}}\check{\mathbf{v}}_\tau$, then, it is clear that $\check{\mathbf{v}}_\tau$ is approximatively orthogonal to $\check{\mathbf{v}}_{\tau'}$, for $\tau' \neq \tau$. By performing the changing variable $\check{\mathbf{v}}_\tau = \mathbf{\Gamma}^{\frac{1}{2}}\mathbf{W}^{\mathrm{H}}\mathbf{v}_\tau$, the desired direction can be selected as the one that is orthogonal to the range space of $\mathcal{F}_{\tau-1}$. To sum up, our algorithm consists in the following steps:

1. Computation of the autocorrelation matrix:

$$\widehat{\mathbf{R}}_K = \sum_{n=1}^{K} \mathbf{x}_N(n)\mathbf{x}_N^{\mathrm{H}}(n).$$

   The signal subspace is estimated by extracting the first d eigenvectors of $\widehat{\mathbf{R}}$.

$$(\mathbf{W}, \mathbf{\Gamma}) = \mathrm{Eig}(\widehat{\mathbf{R}}_K, d).$$

2. Estimation of $\mathbf{T}_\tau = \mathbf{R}_\tau\mathbf{W}\mathbf{\Gamma}^{-\frac{1}{2}}$, and $\mathbf{E} = [\mathbf{e}_0, \cdots, \mathbf{e}_\tau]$, the τ+1 least eigenvectors of $\mathbf{T}_\tau^{\mathrm{H}}\mathbf{T}_\tau = \mathbf{\Gamma}^{-\frac{1}{2}}\mathbf{W}^{\mathrm{H}}\mathbf{R}_\tau^{\mathrm{H}}\mathbf{R}_\tau\mathbf{W}\mathbf{\Gamma}^{-\frac{1}{2}}$.

3. Estimation of $\mathbf{T}_{\tau-1} = \mathbf{R}_{\tau-1}\mathbf{W}\mathbf{\Gamma}^{-\frac{1}{2}}$, and $\mathbf{F} = [\mathbf{f}_0, \cdots, \mathbf{f}_{\tau-1}]$, the τ least eigenvectors of $\mathbf{T}_{\tau-1}^{\mathrm{H}}\mathbf{T}_{\tau-1} = \mathbf{\Gamma}^{-\frac{1}{2}}\mathbf{W}^{\mathrm{H}}\mathbf{R}_{\tau-1}^{\mathrm{H}}\mathbf{R}_{\tau-1}\mathbf{W}\mathbf{\Gamma}^{-\frac{1}{2}}$.

4. Selection of the direction $\check{\mathbf{v}}_\tau$ given by the one that is contained in the range of $\mathbf{E}$ and that is orthogonal to the range of $\mathbf{F}$. More explicitly, $\check{\mathbf{v}}_\tau$ is the least eigenvector of $\mathbf{E}^{\mathrm{H}}\mathbf{F}\mathbf{F}^{\mathrm{H}}\mathbf{E}$. The τ delay equalizer is therefore given by:

$$\mathbf{v}_\tau = \mathbf{W}\mathbf{\Gamma}^{-\frac{1}{2}}\check{\mathbf{v}}_\tau.$$

Remark 1. *If $\tau = 0$, our algorithm will be reduced to the following steps:*

- *Computation of the autocorrelation matrix.*
- *Estimation of $\mathbf{T}_0 = \mathbf{R}_0\mathbf{W}\mathbf{\Gamma}^{-\frac{1}{2}}$ and $\check{\mathbf{v}}_0$ the least eigenvector of $\mathbf{T}_0^{\mathrm{H}}\mathbf{T}_0$. The zero delay equalizer is therefore given by:*

$$\mathbf{v}_0 = \mathbf{W}^{\mathrm{H}}\mathbf{\Gamma}^{-\frac{1}{2}}\check{\mathbf{v}}_0.$$

Remark 2. *In case of channel over-modeling, the matrix $\mathbf{\Gamma}^{-\frac{1}{2}}$ plays an important role in ensuring the robustness of the proposed method. Actually, at moderate or high SNR, it discards the eigenvectors that lie in the noise subspace because they are weighted by $\frac{1}{\sigma_b}$.*

Remark 3. *In this work, we did not seek efficiency in terms of computational cost. Indeed, we can use the relation between $\mathbf{T}_\tau$ and $\mathbf{T}_{\tau-1}$ to eliminate one of the SVDs and reduce the algorithm's complexity. Also, as in [5], we can use efficient subspace tracking techniques in an adaptive scheme to reach linear or close to linear complexity per iteration.*

5. SIMULATIONS

In all our simulations, we consider a SIMO model with $M = 4$ receiving antennas and $L+1 = 4$ channel coefficients chosen randomly according to the Rayleigh distribution. The input signal is drawn from the BPSK constellation and the temporal window N is set to 11. We measure the performance equalizer by using the mean square error (MSE) given by:

$$\mathrm{MSE} = 1 - \mathbb{E}\frac{|\widehat{\mathbf{s}}^{\mathrm{H}}\mathbf{s}|^2}{\|\mathbf{s}\|^2\|\widehat{\mathbf{s}}\|^2}.$$

5.1. Sensitivity to the variance of $\mathbf{h}(0)$

In this section, we investigate the sensitivity of our algorithm to the variance of the first channel coefficient. We have noted that for small values of the first channel coefficient, the equalization process tends to estimate the $\tau+1$ delay signal sequence (rather than the τ delay signal sequence). Like CMA blind methods, we can assume that our method estimates the

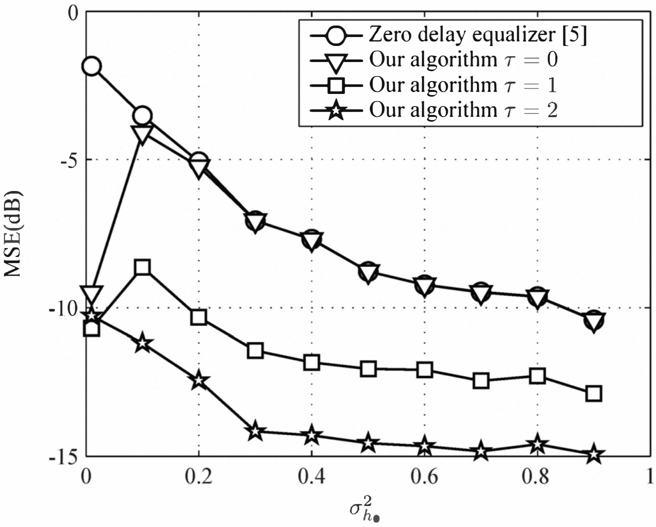

Fig. 1. Sensitivity to the variance of $\mathbf{h}(0)$.

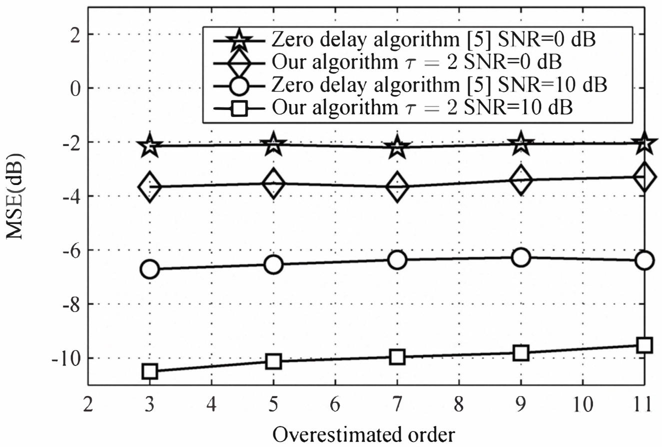

Fig. 2. Evaluation of the robustness of the proposed algorithm.

transmitted sequence up to a certain unknown delay. In practice, this issue can be dealt with by employing a synchronization process and hence we assume that the effective delay that minimizes the errors between the equalized signal and the delay source signal sequences is properly estimated.

Moreover, we expect that even for a non zero delay equalizer, our method should exhibit some sensitivity towards the variance of the first channel coefficient, because the matrices $\mathbf{T}_\tau$ and $\mathbf{T}_{\tau-1}$ may have low singular values thus introducing wrong directions.

Fig. 1 displays the MSE of our algorithm, and compares it with that of the algorithm in [5]. We represent for different values of τ (the equalizer delay) the MSE with respect to the variance of the first channel $\sigma^2_{h_0}$. We note that when the variance of $\mathbf{h}(0)$ is too low, the proposed algorithm exhibits a small degradation in the mean square error performance, as compared to the algorithm in [5]. Besides, for $\tau = 0$ and very low channel coefficient variance $\sigma^2_{\mathbf{h_0}} = 0.01$, our algorithm is able to switch to the delay $\tau = 1$, thus explaining its relatively good performance in this case.

5.2. Robustness to channel order over-modeling

We investigate in this section the robustness of our algorithm to channel order over-modeling. Fig. 2 compares the MSE with respect to the estimated channel order for the proposed equalization process (when $\tau = 2$) and the zero-delay equalizer that is proposed in [5], when the SNR is set to 0 dB and 10 dB, respectively. We note that like the zero delay equalizer, our algorithm is robust to the channel order overestimation, even at low SNR values.

5.3. Effect of the delay

In this section, we investigate the effect of the equalizer delay on the MSE performance of our algorithm. Fig. 3 displays the MSE with respect to the equalizer delay at different SNR values. We can see that almost a gain of 6 dB in MSE can be otained by increasing the equalizer delay. Moreover, as expected, when τ is set above $L + 1 = 4$, the performance enhancement is not significant, in other words, the wide range of gain is approximately achieved when $\tau \geq L + 1$.

5.4. Iterative decoding

As we have previously mentioned, the proposed algorithm is a bit sensitive to the variance of the first channel coefficient (see Fig. 1). In order to further enhance the bit error rate performance, we perform an iterative decoding that makes use of the estimated a posteriori probabilities. Fig. 4 summarizes the iterative process. Given the τ delay equalizer estimate, we compute the variance of the noise $\widehat{\sigma}^2_n = 1 - \widehat{\mathbf{v}}^{\mathrm{H}}_\tau \widehat{\mathbf{R}}_K \mathbf{v}_\tau$. We also estimate the transmitted signal sequence $\mathbf{s}_{n-\tau} = \mathbf{v}^{\mathrm{H}}_\tau \mathbf{X}_n$. For BPSK constellation, the a posteriori probability on the transmitted bits can be easily shown to have the following expression:

$$p_n = \mathrm{APP}(s_n = 1) \triangleq \mathrm{P}(s_n = 1|\widehat{s}_n) = \frac{1}{1 + \exp(-\frac{2\widehat{s}_n}{\widehat{\sigma}^2_n})}.$$

To estimate the vector $\mathbf{g}_\tau$, one can perform hard decision on the equalized signal, as in [5]. But this will not be optimal in the sense that the non reliable entries will have the same contribution as the reliable ones. Using the aposteriori

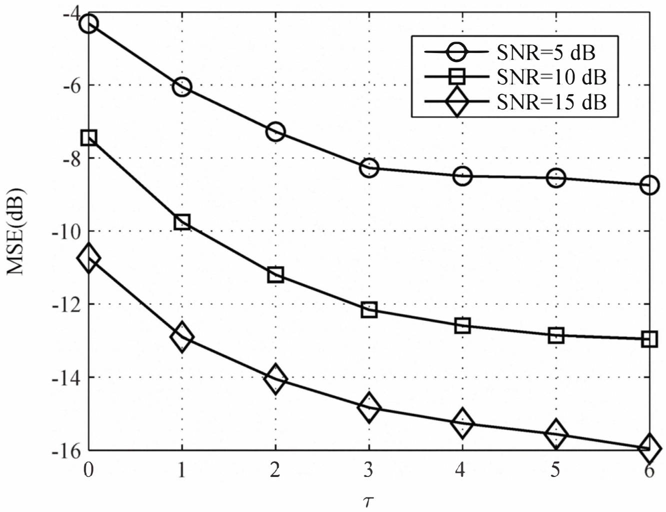

Fig. 3. MSE with respect to the equalizer delay for different SNR values.

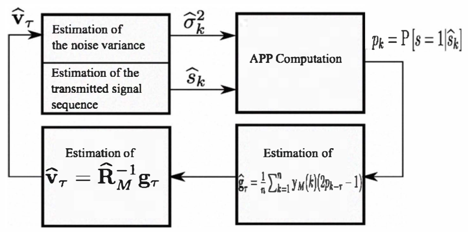

Fig. 4. Iterative decoding for BPSK constellation.

probabilities, the vector $\mathbf{g}_\tau$ can be estimated as:

$$\begin{aligned}\widehat{\mathbf{g}}_\tau &= \frac{1}{K}\sum_{n=1}^{K}\mathbf{X}_n\mathbb{E}\left[\widehat{s}_{n-\tau}\right]\\ &= \frac{1}{K}\sum_{n=1}^{K}\mathbf{X}_n\left(2p_{n-\tau}-1\right).\end{aligned}$$

Fig. 5 displays the bit error rate performance of our algorithm for $\tau = 6$ with the soft iterative. In the legend 'ideal MMSE' refers to the genie MMSE equalizer which exactly knows the correlation matrix $\mathbf{R}$ and the correlation vector $\mathbf{g}_\tau$. We note that with soft iterative processing, the performance of our algorithm can become very close to that of the ideal receiver.

6. CONCLUSION

In this paper, we have generalized a recently proposed zero delay blind equalization to arbitrary delay equalization. Unlike the original method, our technique is much less sensitive to the variance of the first channel coefficient. However, by using an iterative decoding algorithm, our technique can circumvent this issue, and almost reach the performance of an ideal equalizer.

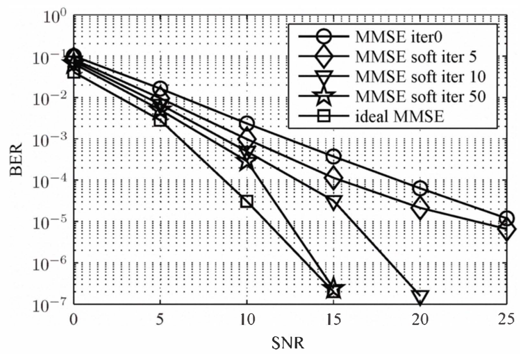

Fig. 5. Bit error rate with soft iterative processing.

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