A decentralized collaborative receive beamforming technique for wireless sensor networks

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***Abstract*—A collaborative receive beamforming technique is proposed that lends itself to a distributed implementation in wireless sensor networks (WSNs). In the first time slot, the transmitter of interest along with several interfering terminals send their signals while in the second time slot K nodes multiply their received signals with properly selected beamforming weights and forward the resulting signals to the receiver. The beamforming weights are selected such that they depend only on the locally-available knowledge at the relaying nodes while aiming to maximize the signal-to-interference-plus-noise ratio (SINR) at the receiver. The beampattern expression is obtained and its analytical properties are studied.**

***Index Terms*—Collaborative beamforming, Wireless sensor networks, Beampattern.**

I. Introduction

Recently, the collaborative beamforming technique has attracted a great interest as a promising solution for a reliable and energy-efficient communication in WSNs [1], [2]. The basic idea behind this technique is that each WSN node collaborates in the transmission by sending a properly scaled version of its received signal [3]. When the total transmit power from the K collaborating nodes is fixed, the transmit power from each node decreases inversely proportional to K and, at the same time, the SINR linearly increases with K [1]. Despite this significant practical merit, to perform a collaborative beamforming the collaborating nodes must be aware of the local information such as locations and forward channels of each others. However, due to the distributed feature of WSNs, the nodes are independent units and, hence, they cannot satisfy the latter requirement [4], [5].

In this paper, we propose a novel collaborative beamforming technique that can be implemented in distributed networks. We consider a scenario wherein a set of transmitters that comprised of a source and several interferences send their signals to a receiver using a WSN with uniformly distributed nodes. To achieve a reliable communication, we properly select the beamforming weights that maximize the SINR at the receiver subject to the WSN nodes total transmit power constraint. The optimal weight of each node turns out to be dependent on information that is not available at that node. This renders the so-obtained SINR-optimal beamformer not suitable for an implementation in a distributed network. Using the fact that the number of collaborating nodes is typically large in practice, we propose a new collaborative beamformer whose weights depend only on the information commonly available at every node and, further, well-approximates its SINR-optimal beamformer counterpart. We then derive the average beampattern expression of the proposed decentralized collaborative beamformer and study its properties to verify the effectiveness of the proposed technique in terms of increasing the desired received power and suppressing the interferences.

The rest of this paper is organized as follows. The system model is presented in Section II and the SINR-optimal beamformer is derived in Section III. Section IV proposes the decentralized collaborative beamformer and Section V derives the average beampattern expression and analyzes its characteristics. Simulation results are presented in section VI and concluding remarks are given in Section VII.

Notation: Uppercase and lowercase bold letters denote matrices and vectors, respectively. $[\cdot]_{il}$ and $[\cdot]_i$ are the (i,l)-th entry of a matrix and i-th entry of a vector, respectively. $\mathbf{I}$ is the identity matrix. $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the Hermitian transpose, respectively. $\|\cdot\|$ is the 2-norm of a vector and $|\cdot|$ is the absolute value. $\mathrm{E}\{\cdot\}$ stands for the statistical expectation and $(\xrightarrow{ep1})$ $\xrightarrow{p1}$ denotes (element-wise) convergence with probability one. $J_1(\cdot)$ is the first order Bessel function of the first kind and $\odot$ is the element-wise product.

II. System model

Fig. 1 depicts the system of our concern that is comprised of one receiving node at O, K uniformly distributed nodes on $D(O,R)$, the disc of radius R centered at O, and L transmitters located in the far field. The transmitted signal from the first transmitter is the desired signal while the other $L-1$ transmitters act as interferences. Let O be the pole and the ray towards the desired transmitter be the polar axis of a polar coordinate system and denote the coordinates of the k-th node as (r_k,ψ_k) and those of the l-th transmitter as (A_l,ϕ_l) where $\phi_1=0$ and $A_l \gg R$ for $l=1,\ldots,L$. The following common assumptions are also used:

A1) Scattering and reflection effects from the transmitters to the nodes are negligible [1]-[4]. The channel gain from the l-th transmitter to the k-th node can be then represented as $[\mathbf{g}_l]_k = c_l e^{j(2\pi/\lambda)d_{kl}}$ where c_l is the signal path-loss, λ is the wavelength, and d_{kl} is the distance between the two terminals. Using the fact that $A_l \gg R$ and, hence, $d_{kl} \approx A_l - r_k\cos(\psi_k-\phi_l)$, $[\mathbf{g}_l]_k$ can be approximated by $b_l[\bar{\mathbf{g}}_l]_k$ with $[\bar{\mathbf{g}}_l]_k \triangleq e^{-j(2\pi/\lambda)r_k\cos(\psi_k-\phi_l)}$ and $b_l \triangleq c_l e^{j(2\pi/\lambda)A_l}$ [1].

A2) The channel gain $[\mathbf{f}]_k$ between the k-th node and the receiver is a zero-mean unit-variance circular Gaussian random variable.

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A3) Transmitted signals $s_l,\ l=1,\ldots,L$ are zero-mean identically distributed random variables while noises at nodes and the receiver are zero-mean Gaussian random variables with variances ${\sigma_v}^2$ and ${\sigma_n}^2$, respectively. All signals, noises, and the nodes forward channel gains are mutually independent.

A4) The k-th node is aware of its own coordinates (r_k, ψ_k), its forward channel $[\mathbf{f}]_k$, the directions of the transmitting terminals $\phi_l,\ l=1,\ldots,L$, and the nodes' permitted total transmit power $P_{\max}$, σ_v^2, and σ_n^2, while being oblivious to the locations and the forward channels of *all* other nodes in the network.

A1-A3 are common in the literature of array processing for planar waves and are frequently adopted in the context of collaborative beamforming for WSNs. In particular, A1 is reasonable since the transmitters' distances from WSN are much larger than the size of WSN. In such a case, the large-scale fading plays the dominant role [2]. In turn, A2 is valid when R is relatively small, and, therefore, the small scale fading effects are dominant [2]. Finally, note that A4 is due to the distributed and decentralized feature of WSNs.

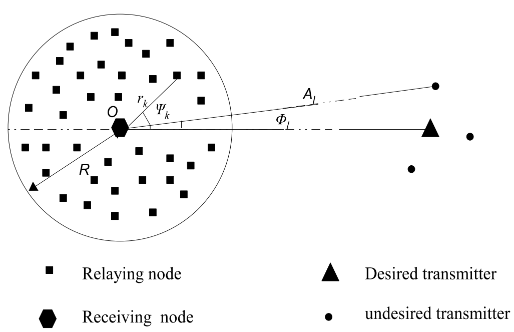

Fig. 1. System model.

III. SINR-OPTIMAL BEAMFORMER

The transmission from the L transmitters to the receiver is performed in two time slots. All transmitters send their signals in the first time slot, while each node k multiplies its received signal by a beamforming weight w_k and forwards it to the receiver in the second time slot. It can be readily shown that the received signal at O is given by

$$r = b_1 s_1 \mathbf{w}^H \mathbf{h}_1 + \mathbf{w}^H \mathbf{H}_{\bar{1}} \mathbf{B} \mathbf{s}_{\bar{1}} + \mathbf{w}^H(\mathbf{f} \odot \mathbf{v}) + n \qquad (1)$$

where $\mathbf{w} \triangleq [\mathbf{w}_1 \ldots \mathbf{w}_K]$ is the beamforming vector, n and v are respectively the receiver and nodes' noises, $\mathbf{B} \triangleq \text{diag}\{b_2,\ldots,b_L\}$, $\mathbf{s}_{\bar{1}} \triangleq [s_2 \ldots s_L]^T$, $\mathbf{h}_1 \triangleq \mathbf{f} \odot \bar{\mathbf{g}}_1$ and $\mathbf{H}_{\bar{1}} \triangleq [\mathbf{f} \odot \bar{\mathbf{g}}_2 \ldots \mathbf{f} \odot \bar{\mathbf{g}}_L]$ with $\bar{\mathbf{g}}_l \triangleq [[\bar{\mathbf{g}}_l]_1 \ldots [\bar{\mathbf{g}}_l]_K]^T$ for $l=1,\ldots,L$ and $\mathbf{f} \triangleq [[\mathbf{f}]_1 \ldots [\mathbf{f}]_K]^T$.

It follows from (1) that the received power from the source, the received power from the interferences, and the aggregate noise power due to the thermal noise at the receiver and the forwarded noise from the beamforming nodes can be respectively written as

$$P_S = p_1 |b_1 \mathbf{w}^H \mathbf{h}_1|^2 \qquad (2)$$
$$P_I = \mathbf{w}^H \mathbf{H}_{\bar{1}} \mathbf{B} \mathbf{P}_{\bar{1}} \mathbf{B}^H \mathbf{H}_{\bar{1}}^H \mathbf{w} \qquad (3)$$
$$P_N = \mathbf{w}^H \mathbf{\Sigma} \mathbf{w} + \sigma_n^2 \qquad (4)$$

where $\mathbf{\Sigma} \triangleq \sigma_v^2 \text{diag}\{|[\mathbf{f}]_1|^2,\ldots,|[\mathbf{f}]_k|^2\}$, p_1 is the source signal power, and $\mathbf{P}_{\bar{1}} \triangleq \text{diag}\{p_2,\ldots,p_L\}$ with $p_l,\ l=2,\ldots,L$ being the l-th transmitter signal power. It directly follows from (2)-(4) that the SINR at the receiver is given by

$$\text{SINR} = \frac{p_1 |b_1 \mathbf{w}^H \mathbf{h}_1|^2}{\mathbf{w}^H \left(\mathbf{H}_{\bar{1}} \mathbf{B} \mathbf{P}_{\bar{1}} \mathbf{B}^H \mathbf{H}_{\bar{1}}^H + \mathbf{\Sigma}\right) \mathbf{w} + \sigma_n^2}. \qquad (5)$$

To achieve a reliable transmission, the objective is to find a beamforming vector $\mathbf{w}_{\text{opt}}$ that maximizes the SINR subject to the nodes total transmit power constraint. It can be proved that $\mathbf{w}_{\text{opt}}$ is given by

$$\mathbf{w}_{\text{opt}} = \mu \left(\mathbf{\Lambda}^{-1} \mathbf{h}_1 - \mathbf{\Lambda}^{-1} \mathbf{H}_{\bar{1}} \mathbf{c}\right) \qquad (6)$$

where

$$\mathbf{c} \triangleq \left(\left(\mathbf{B} \mathbf{P}_{\bar{1}} \mathbf{B}^H\right)^{-1} + \mathbf{H}_{\bar{1}}^H \mathbf{\Lambda}^{-1} \mathbf{H}_{\bar{1}}\right)^{-1} \mathbf{H}_{\bar{1}}^H \mathbf{\Lambda}^{-1} \mathbf{h}_1, \qquad (7)$$

$\mathbf{\Lambda} \triangleq \mathbf{\Sigma} + \left(\sigma_n^2 B / P_{\max}\right) \mathbf{I}$ with $B = \sigma_v^2 + \sum_{l=1}^L |b_l|^2 p_l$, and

$$\mu = \left(\frac{P_{\max}}{B\left(\mathbf{h}_1^H \mathbf{\Lambda}^{-2} \mathbf{h}_1 - \mathbf{d}^H \mathbf{c} - \mathbf{c}^H \mathbf{d} + \mathbf{c}^H \mathbf{D} \mathbf{c}\right)}\right)^{1/2} \qquad (8)$$

with $\mathbf{d} = \mathbf{H}_{\bar{1}}^H \mathbf{\Lambda}^{-2} \mathbf{h}_1$ and $\mathbf{D} = \mathbf{H}_{\bar{1}}^H \mathbf{\Lambda}^{-2} \mathbf{H}_{\bar{1}}$. Note that μ is chosen such that the nodes total transmit power is equal to $P_{\max}$.

The beamforming vector $\mathbf{w}_{\text{opt}}$ is implementable only if the k-th node can compute its corresponding beamforming weight

$$[\mathbf{w}_{\text{opt}}]_k = \mu \left([\mathbf{\Lambda}]_{kk}^{-1} [\mathbf{h}_1]_k - [\mathbf{\Lambda}]_{kk}^{-1} \sum_{l=1}^{L-1} [\mathbf{H}_{\bar{1}}]_{kl} [\mathbf{c}]_l\right). \qquad (9)$$

As can be observed from (9), μ, $[\mathbf{\Lambda}]_{kk}$, $[\mathbf{h}_1]_k$, the k-th row of $\mathbf{H}_{\bar{1}}$, and all entries of $\mathbf{c}$ must be locally computable at the k-th node. As $[\mathbf{\Lambda}]_{kk}$, $[\mathbf{h}_1]_k$ and the k-th row of $\mathbf{H}_{\bar{1}}$ depend only on the information available at the kth node, they satisfy the latter requirement. However, μ and the entries of $\mathbf{c}$ are functions of all nodes' locations and forward channels and, according to A4, they cannot be computed at each node. Therefore, although $\mathbf{w}_{\text{opt}}$ maximizes the SINR, it cannot be implemented in the distributed network of our concern. A new decentralized collaborative beamformer is proposed in Section IV that well-approximates $\mathbf{w}_{\text{opt}}$ and, further, its weights depend solely on the information available at the corresponding nodes.

IV. PROPOSED DECENTRALIZED COLLABORATIVE BEAMFORMER

An approach to get around the problem exposed in the previous section is to substitute $\mathbf{c}$ and μ with quantities that not only can be computed at each individual node, but also well-approximate their original counterparts. Note that when the nodes total transmit power is fixed, the transmit power from each node is inversely proportional to K and the SINR linearly increases with K. Therefore, it makes practical sense to use a larger K in the collaborative beamforming procedure. In such a case, $\mathbf{c}$ and μ can be substituted with $\tilde{\mathbf{c}} \triangleq \lim_{K\to\infty} \mathbf{c}$ and $\tilde{\mu} \triangleq \lim_{K\to\infty} \mu$, respectively. While $\tilde{\mathbf{c}}$ and $\tilde{\mu}$ are good approximations of their original counterparts, they must also solely depend on the information commonly available at all the nodes. In what follows, we will prove that these approximations satisfy the latter requirement.

As $\lim_{K\to\infty} 1/\left(K\,|b_l|^2\,p_l\right) = 0$, it can be shown that

$$\tilde{\mathbf{c}} = \left(\lim_{K\to\infty} \frac{1}{K}\mathbf{H}_{\bar{1}}^H\mathbf{\Lambda}^{-1}\mathbf{H}_{\bar{1}}\right)^{-1}\cdot\left(\lim_{K\to\infty} \frac{1}{K}\mathbf{H}_{\bar{1}}^H\mathbf{\Lambda}^{-1}\mathbf{h}_1\right). \quad (10)$$

First, let us derive the limit of the first expression in the right-hand side (RHS) of (10). It follows from the definitions of $\mathbf{H}_{\bar{1}}$ and $\mathbf{\Lambda}$ that

$$[\mathbf{H}_{\bar{1}}^H\mathbf{\Lambda}^{-1}\mathbf{H}_{\bar{1}}]_{mn} = \sum_{k=1}^{K} \frac{|[\mathbf{f}]_k|^2\, e^{j\alpha(\phi_{m+1}-\phi_{n+1})z_k}}{\sigma_v^2\,|[\mathbf{f}]_k|^2 + \frac{\sigma_n^2 B}{P_{\max}}} \quad (11)$$

where $z_k \triangleq r_k \sin\left(\psi_k - (\phi_{m+1}+\phi_{n+1})/2\right)/R$ and $\alpha(\phi) \triangleq 4\pi R\sin(\phi/2)/\lambda$. Using the strong law of large numbers and the fact that $[\mathbf{f}]_k$, r_k and ψ_k are all mutually statistically independent and the nodes are uniformly distributed on $D(O,R)$, we obtain that

$$\lim_{K\to\infty} \frac{1}{K}[\mathbf{H}_{\bar{1}}^H\mathbf{\Lambda}^{-1}\mathbf{H}_{\bar{1}}]_{mn} \xrightarrow{p1} 2q\mathbf{E} \quad (12)$$

where $\mathbf{E}$ is an $L-1\times L-1$ matrix with

$$[\mathbf{E}]_{mn} \triangleq \begin{cases} \frac{J_1(\alpha(\phi_{m+1}-\phi_{n+1}))}{\alpha(\phi_{m+1}-\phi_{n+1})} & m \neq n \\ \frac{1}{2} & m = n \end{cases} \quad (13)$$

and

$$\begin{aligned} q &\triangleq \mathrm{E}\left\{\frac{|[\mathbf{f}]_k|^2}{\sigma_v^2\,|[\mathbf{f}]_k|^2 + \frac{\sigma_n^2 B}{P_{\max}}}\right\} \\ &= \frac{1}{\sigma_v^2}\int_0^\infty \frac{xe^{-x}dx}{x+\varrho} \\ &= \frac{1}{\sigma_v^2}\left(\varrho e^{\varrho}E_i(-\varrho)+1\right). \end{aligned} \quad (14)$$

In (14), $\varrho = \sigma_n^2 B/\sigma_v^2 P_{\max}$ and $E_i(x) = \int_{-\infty}^{x}(e^t/t)dt$ is the exponential integral function. Following a similar approach as in (11), it can also be shown that

$$\lim_{K\to\infty} \frac{1}{K}\left[\mathbf{H}_{\bar{1}}^H\mathbf{\Lambda}^{-1}\mathbf{h}_1\right]_m \xrightarrow{p1} 2q\,[\mathbf{z}]_m \quad (15)$$

where $\mathbf{z}$ is an $L-1\times 1$ vector with

$$[\mathbf{z}]_m \triangleq \begin{cases} \frac{J_1(\alpha(\phi_{m+1}))}{\alpha(\phi_{m+1})} & \phi_{m+1} \neq 0 \\ \frac{1}{2} & \phi_{m+1} = 0 \end{cases}. \quad (16)$$

Therefore, it follows from (12) and (15) that

$$\tilde{\mathbf{c}} \xrightarrow{ep1} \mathbf{E}^{-1}\mathbf{z}. \quad (17)$$

Equations (13) and (16) show that $\mathbf{E}$ and $\mathbf{z}$ depend only on the directions of the interferences and, hence, according to A4, all the entries of $\tilde{\mathbf{c}}$ can be locally computed at each individual node.

Now, let us turn our attention to compute $\tilde{\mu}$. Using (13) and (16), it can be shown that

$$\lim_{K\to\infty} \frac{1}{K}\left(\mathbf{h}_1^H\mathbf{\Lambda}^{-2}\mathbf{h}_1\right) \xrightarrow{p1} \tilde{q} \quad (18)$$

$$\lim_{K\to\infty} \frac{1}{K}\left[\mathbf{d}^H\right]_m \xrightarrow{p1} 2\tilde{q}\left[\mathbf{z}^T\right]_m \quad (19)$$

$$\lim_{K\to\infty} \frac{1}{K}[\mathbf{D}]_{mn} \xrightarrow{p1} 2\tilde{q}[\mathbf{E}]_{mn} \quad (20)$$

$$\lim_{K\to\infty} \frac{1}{K}[\mathbf{d}]_m \xrightarrow{p1} 2\tilde{q}[\mathbf{z}]_m \quad (21)$$

where

$$\begin{aligned} \tilde{q} &\triangleq \mathrm{E}\left\{\frac{|[\mathbf{f}]_k|^2}{\left(\sigma_v^2\,|[\mathbf{f}]_k|^2 + \frac{\sigma_n^2 B}{P_{\max}}\right)^2}\right\} \\ &= \frac{1}{\sigma_v^4}\int_0^\infty \frac{xe^{-x}dx}{(x+\varrho)^2} \\ &= \frac{-1}{\sigma_v^4}\left(e^{\varrho}E_i(-\varrho)(1+\varrho)+1\right). \end{aligned} \quad (22)$$

Thus, $\tilde{\mu}$ can be expressed as

$$\tilde{\mu} \xrightarrow{p1} \left(\frac{P_{\max}}{K\tilde{q}B\left(1-2\mathbf{z}^T\mathbf{E}^{-1}\mathbf{z}\right)}\right)^{1/2}. \quad (23)$$

It can be observed from (23) that $\tilde{\mu}$ does not depend on the locations and the forward channels of any nodes and, therefore, it is locally computable at each individual node. Using (17) and (23) we introduce

$$\tilde{\mathbf{w}} = \tilde{\mu}\left(\mathbf{\Lambda}^{-1}\mathbf{h}_1 - \mathbf{\Lambda}^{-1}\mathbf{H}_{\bar{1}}\mathbf{E}^{-1}\mathbf{z}\right). \quad (24)$$

The above beamforming vector not only can be implemented in a decentralized fashion, but also well-approximates its optimal counterpart $\mathbf{w}_{\mathrm{opt}}$ for a large K. In Section V, we derive the average beampattern expression associated with $\tilde{\mathbf{w}}$ and study its proprieties.

V. Average Beampattern

Beampattern is the received power from a transmitter at an arbitrary location $(A_\star, \phi_\star)$. Therefore, it is a performance measure that can evaluate the efficiency of the beamformer in increasing the received power from the source at $(A_1, \phi_1 = 0)$ and reducing the received power from the interfering terminals at (A_l, ϕ_l) $l = 2,\ldots,L$.

Assuming that the transmitter at $(A_\star, \phi_\star)$ has the transmit power of $p_\star$, the beampattern associated with our proposed beamformer is given by

$$P(A_\star, \phi_\star) = p_\star\left|\tilde{\mathbf{w}}^H(\mathbf{f}\odot\mathbf{g}_\star)\right|^2 \quad (25)$$

where $\mathbf{g}_\star \triangleq [[\mathbf{g}_\star]_1 \ldots [\mathbf{g}_\star]_K]^T$ is the channel gain vector with

$$[\mathbf{g}_\star]_k \triangleq b_\star e^{-j(2\pi/\lambda)r_k\cos(\psi_k-\phi_\star)}. \quad (26)$$

In (26), $b_\star \triangleq c_\star e^{j(2\pi/\lambda)A_\star}$ and $c_\star$ is the path-loss.

To verify the efficiency of the proposed beamforming technique, we need to show that the beampattern has a narrow mainbeam centered at $(A_1, \phi_1 = 0)$ and minima at (A_l, ϕ_l) $l = 2, \ldots, L$. As $P(A_\star, \phi_\star)$ is a function of the random variables $r_k, \psi_k, [\mathbf{f}]_k$ $k = 1, \ldots, K$, it is more practical to investigate the behavior of the average beampattern $P_{\rm av}(A_\star, \phi_\star) = {\rm E}\{P(A_\star, \phi_\star)\}$ where the expectation is taken with respect to the nodes' coordinates and their forward channels. It can be shown that [6]

$$P_{\rm av}(A_\star, \phi_\star) = \Omega(A_\star)\Theta(\phi_\star) \tag{27}$$

where

$$\Omega(A_\star) = \frac{|b_\star|^2 p_\star P_{\max}}{\tilde{q}B} \tag{28}$$

and

$$\Theta(\phi_\star) = p + \frac{4(K-1)q^2}{(1 - 2\mathbf{z}^T\mathbf{E}^{-1}\mathbf{z})}\left(\frac{J_1(\alpha(\phi_\star))}{\alpha(\phi_\star)} - \mathbf{z}_{\phi_\star}^T\mathbf{E}^{-1}\mathbf{z}\right)^2 \tag{29}$$

with $\mathbf{z}_{\phi_\star}$ being an $L-1 \times 1$ vector such that

$$\left[\mathbf{z}_{\phi_\star}\right]_l \triangleq \begin{cases} \frac{J_1(\alpha(\phi_\star - \phi_{l+1}))}{\alpha(\phi_\star - \phi_{l+1})} & \phi_\star \neq \phi_{l+1} \\ \frac{1}{2} & \phi_\star = \phi_{l+1} \end{cases} \tag{30}$$

and

$$\begin{aligned} p &= {\rm E}\left\{\frac{|[\mathbf{f}]_k|^4}{\left(\sigma_v^2\,|[\mathbf{f}]_k|^2 + \frac{\sigma_n^2 B}{P_{\max}}\right)^2}\right\} \\ &= \frac{1}{\sigma_v^4}\int_0^\infty \frac{x^2 e^{-x} dx}{(x+\varrho)^2} \\ &= \frac{1}{\sigma_v^4}\left(\varrho e^{\varrho} E_i(-\varrho)(2+\varrho) + \varrho + 1\right). \end{aligned} \tag{31}$$

It follows from (27) that the average beampattern value at the location of the desired source is

$$P_{\rm av}(A_1, \phi_1 = 0) = \Omega(A_1)\left(p + (K-1)q^2\big(1 - 2\mathbf{z}^T\mathbf{E}^{-1}\mathbf{z}\big)\right). \tag{32}$$

In turn, it can be readily proved that the average beampattern at the location of the l-th interference is [6]

$$P_{\rm av}(A_l, \phi_l) = \frac{|b_l|^2 p_l P_{\max} p}{\tilde{q}B}. \tag{33}$$

As can be observed from (32) and (33), the received power from the desired source linearly increases with K while the received power from the interferences remains constant. This verify the efficiency of the proposed beamformer in increasing the desired signal power and, hence, the SINR. It can also be shown that [6]

$$\lim_{K\to\infty} P(A_\star, \phi_\star) \xrightarrow{p1} \lim_{K\to\infty} P_{\rm av}(A_\star, \phi_\star). \tag{34}$$

Equation (34) implies that as long as K is large enough, any realization of the beampattern has similar proprieties as the average beampattern.

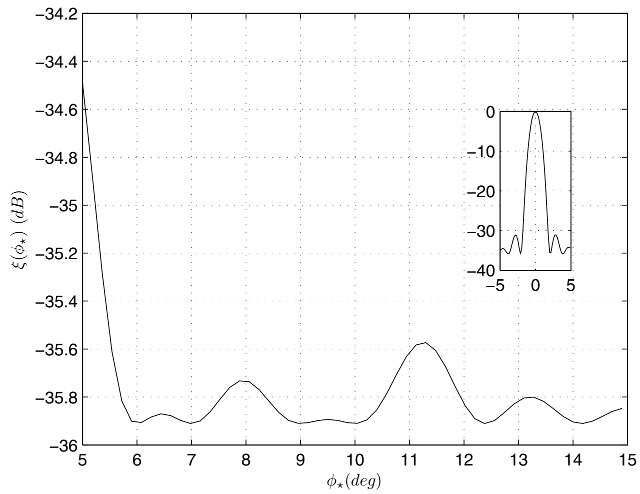

Fig. 2. $\xi(\phi_\star)$ in the case that $[\phi_2, \phi_3, \phi_4, \phi_5] = [6, 7, 9, 10](deg)$.

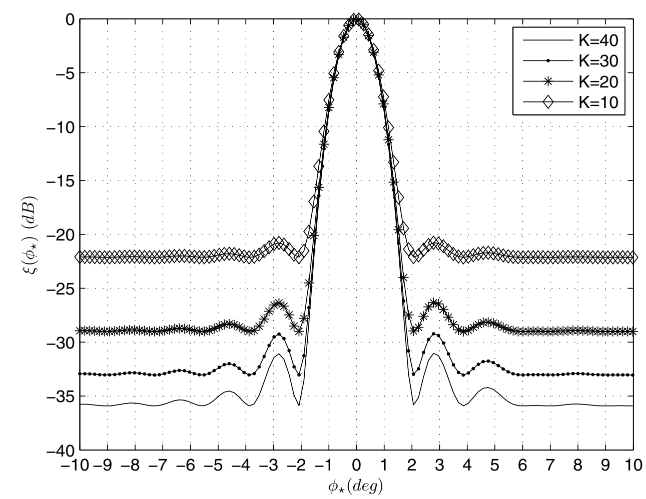

Fig. 3. $\xi(\phi_\star)$ for $K = 10, 20, 30, 40$.

VI. Simulation

Computer simulations are used to validate the analytical results. We consider $L = 5$ and assume that all transmitters have equal powers and that the noises' powers σ_n^2 and σ_v^2 are $20dB$ below the nodes' permitted total transmit power $P_{\max}$. Fig. 3 shows $\xi(\phi_\star) = \Theta(\phi_\star)/\Theta(0)$ in the case that $[\phi_2, \phi_3, \phi_4, \phi_5] = [6, 7, 9, 10](deg)$. As can be observed from this figure, the beampattern has a narrow mainlobe with a peak at $\phi_1 = 0$ and four minima at the directions of the interferences. This verifies the beamformer's efficiency in enhancing the desired signal power and suppressing the interferences' powers. Fig. 3 shows $\xi(\phi_\star)$ for $K = 10, 20, 30, 40$. As expected, $\xi(\phi_l)$, $l = 2, \ldots, L$ decreases inversely proportional to K and, thus, the efficiency of the proposed decentralized beamformer is improved as K grows large.

VII. Conclusion

In this work, we have considered a SINR-optimal beamformer. It has been shown that each node needs to be aware of the locations and the forward channels of all other nodes, in order to compute its corresponding optimal weight. Using the fact that the number of nodes is typically large, we have proposed a decentralized collaborative beamformer that well-approximates the SINR-optimal beamformer and, further, can be implemented in a decentralized fashion. Analyzing the average beampattern associated with the proposed beamformer, we conclude that it effectively increases the received power of the source and suppresses those of the interferences. An important topic for future work is to verify the robustness of the proposed decentralized beamformer against inaccuracies in the transmitters' directions estimation and the noise power estimation.

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