Adaptive Preprocessing and Ordering for Detection in MIMO Systems over Rayleigh Fading Channels

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***Abstract*—In this paper, considering a multiple-input multiple-output (MIMO) system, we expose an adaptive method for a nulling-and-cancelling detection scheme. The proposed method takes advantage of the temporal correlation of a Rayleigh fading MIMO channel in order to reduce the complexity of all the decoding steps. We especially focus on the decoding ordering, which is a crucial step in a zero forcing detection feedback equalizer (ZF-DFE) decoding scheme. The corresponding columns permutation performed on the channel matrix is adaptively updated from the past channel information using an ad-hoc fixed parameter that offers a very good performance/complexity tradeoff.**

***Index Terms*—mulitple-input multiple-output (MIMO), Bell Labs layered space-time (BLAST), Rayleigh fading channel, adaptive detection, detection ordering**

I. INTRODUCTION

In a wireless fading environnement, the use of multiple antennas is theoretically capable of huge capacities if the multipath scattering is sufficiently rich and appropriately used. One practical transmission scheme over multipath wireless fading channels with multiple input multiple output (MIMO) is the Bell Laboratories layered space-time (BLAST), which can almost achieve the Shannon capacity [1].

In MIMO systems, the outputs are a linear combination of inputs corrupted by an additive noise, and the decoding is a challenging problem. Some techniques previously developped in the litterature give complexities that are polynomial at high signal to noise ratio (SNR) values, but remain exponential at low SNR values. One technique has been recently developped in [2], which gives near maximum-likelihood (ML) solutions and reduces the decoding complexity almost independently of the SNR.

In recent years, different schemes have been proposed and improved to solve this problem and reduce the complexity in BLAST systems decoding schemes [1], [3], [4]. Other researchers have developped techniques from the lattice theory, since the research of the maximum-likilhood (ML) estimate of the transmitted data can be seen as a closest lattice point search (CLPS) problem [5], [6], [7].

In these methods, the complexity required to achieve the optimal performance is basically very high and one wishes this complexity to be reduced. The preprocessing and ordering stage needed for the detection is especially computationally costly. A very efficient remedy is to take advantage of the temporal correlation of the fading channel. One method has been recently published [10] that updates through time a unimodular matrix required for lattice reduction in the CLPS approach.

In this paper, we propose an adaptive method for the preprocessing and ordering stage required for a nulling and cancelling BLAST detection scheme. We focus on the detection ordering, which is a crucial point in this kind of detection.

In this paper, the channel is not assumed to be constant troughout the data frame and the successive channel realizations are temporally correlated. This scenoario corresponds to the cases when the channel is slowly varying or when the frame length over which the channel can be considered constant is quite small. Our method uses the results found for the previous channel realization to update the preprocessing and ordering stage for the new channel realization.

The rest of the paper is organized as follows. Section II presents the system model. Section III expose the non-adaptive preprocessing and ordering stage. An adaptive way to perform it is proposed in Section IV. Some simulation results are shown in Section V and Section VI gives some concluding remarks.

II. SYSTEM MODEL

In this paper, we consider a MIMO system with M transmit and N receive antennas. We also assume a Rayleigh fading channel modeled by a $N \times M$ complex matrix $\mathbf{H}^c$ with independent entries:

$$\mathbf{H}^c = \begin{pmatrix} h^c_{1,1} & h^c_{1,2} & \cdots & h^c_{1,M} \\ h^c_{2,1} & h^c_{2,2} & \cdots & h^c_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ h^c_{N,1} & h^c_{N,2} & \cdots & h^c_{N,M} \end{pmatrix}. \quad (1)$$

Denoting by $\mathbf{x}^c = [x^c_1, x^c_2, ..., x^c_M]^T$ the complex vector of the transmitted signal on the M transmit antennas, $\mathbf{y}^c = [y^c_1, y^c_2, ..., y^c_M]^T$ the complex received signal on the N receiving antennas and $\mathbf{w}^c = [w^c_1, w^c_2, ..., w^c_M]^T$ the complex Gaussian noise vector with zero-mean and variance $\sigma^2\mathbf{I}_N$, we can write:

$$\mathbf{y}^c = \mathbf{H}^c\mathbf{x}^c + \mathbf{w}^c. \quad (2)$$

The complex signal $\mathbf{x}^c$ is drawn from a Q^2-QAM constellation assumed to be normalized, i.e., having unit energy. We transform the system as follows in order to work with real data:

$$\mathbf{H}^c \rightarrow \mathbf{H} = \left[\begin{array}{cc} \Re(\mathbf{H}^c) & -\Im(\mathbf{H}^c) \\ \Im(\mathbf{H}^c) & \Re(\mathbf{H}^c) \end{array} \right], \quad (3)$$

$$\mathbf{u}^c \rightarrow \mathbf{u} = \left[\begin{array}{cc} \Re(\mathbf{u}^c) & \Im(\mathbf{u}^c) \end{array} \right], \quad (4)$$

where $\mathbf{u}^c$ denotes any complex vector. $\Re(.)$ denotes the real part of a complex and $\Im(.)$ its imaginary part.

We now deal with a real system of size $2N \times 2M$. For ease of notation, we will use in the following $m = 2M$ and $n = 2N$.

Each of the components of the transmitted signal vector $\mathbf{x}$ is then drawn from a Q-PAM and we can write:

$$\mathbf{x} = k\mathbf{c} + \mathbf{v}, \quad (5)$$

where the elements of $\mathbf{c}$ are in $\mathcal{U} = \{0, 1, ..., Q-1\}$, $\mathbf{v}$ is a constant vector and k is related to the constellation energy. We can then rewrite (2):

$$\begin{aligned} \mathbf{y} &= \mathbf{Hx} + \mathbf{w}, & (6) \\ &= \mathbf{H}\left(k\mathbf{c} + \mathbf{v}\right) + \mathbf{w}. & (7) \end{aligned}$$

III. Hard-output Detection

The maximum-likelihood (ML) solution to the MIMO system is given by:

$$\widehat{\mathbf{x}}^c = \arg \min_{\mathbf{x}^c \in \chi^m} \|\mathbf{y}^c - \mathbf{H}^c\mathbf{x}^c\|^2, \quad (8)$$

where χ is the set of the Q^2-QAM constellation elements. Using the previous transformations, on can rewrite this minimization in the following way:

$$\begin{aligned} \widehat{\mathbf{c}} &= \arg \min_{\mathbf{c} \in \mathcal{U}^m} \|\mathbf{y} - \mathbf{H}\left(k\mathbf{c} + \mathbf{v}\right)\|^2, & (9) \\ &= \arg \min_{\mathbf{c} \in \mathcal{U}^m} \|\mathbf{y}' - \mathbf{H}'\mathbf{c}\|^2, & (10) \end{aligned}$$

with $\mathbf{y}' = \mathbf{y} - \mathbf{Hv}$ and $\mathbf{H}' = k\mathbf{H}$.

A good approximation of the ML solution is the linear zero forcing (ZF) with decision feedback equalizer (DFE) performed by QR decomposition on $\mathbf{H}'$:

$$\mathbf{H}' = \left[\begin{array}{cc} \mathbf{Q_1} & \mathbf{Q_2} \end{array} \right] \left[\begin{array}{c} \mathbf{R} \\ \mathbf{0} \end{array} \right], \quad (11)$$

where $\mathbf{R}$ is an $m \times m$ upper triangular matrix, $\mathbf{0}$ is a $(n-m) \times n$ zero matrix and $\mathbf{Q_1} and \mathbf{Q_2}$ are unitary matrices of size, respectively, $n \times m$ and $n \times (n-m)$. Then, denoting $\mathbf{y}'' = \mathbf{Q_1}^T\mathbf{y}'$ one can once again rewrite the minimization problem (8):

$$\widehat{\mathbf{c}} = \arg \min_{\mathbf{c} \in \mathcal{U}^m} \|\mathbf{y}'' - \mathbf{Rc}\|^2. \quad (12)$$

The ZF-DFE decoding scheme works as follows. Using the upper triangular shape of $\mathbf{R}$, the estimate $\widehat{c}_m$ of the last symbol c_m is simply obtained by:

$$\widehat{c}_m = \left\lceil \frac{y''_m}{R_{m,m}} \right\rfloor. \quad (13)$$

One can afterwards easily cancel the noise interference due to c_m in $y''_{m-1} = R_{m-1,m-1}c_{m-1} + R_{m-1,m}c_m$ by substituting c_m by its estimate $\widehat{c}_m$ and obtain the estimate of c_{m-1}:

$$\widehat{c}_{m-1} = \left\lceil \frac{y''_{m-1} - R_{m-1,m}\widehat{c}_m}{R_{m-1,m-1}} \right\rfloor. \quad (14)$$

Finally, the solution appears to be:

$$\widehat{c}_i = \left\lceil \frac{y''_i - \sum_{j=i+1}^{m} R_{i,j}\widehat{c}_j}{R_{i,i}} \right\rfloor. \quad (15)$$

A crucial problem in this scheme is error propagation. Indeed, each estimate depends essentially on the previous estimates. If the estimate of c_m is very far from its real value, it will affect all the other estimates. Thus, in order to minimize this effect, one has to choose an efficient decoding order that minimizes the error propagation effect. The optimal order is to decode the symbols by decreasing SNR [1]. However, this requires the computation of the error covariance matrix $\left[\mathbf{H}'^H\mathbf{H} + \alpha\mathbf{I}_m\right]^{-1}$, with $\alpha = 1/SNR$. An other ordering, that offers a satisfying complexity/performance tradeoff is to order the columns of $\mathbf{H}'$ in a non-decreasing order.

An efficient way to improve the error performance of ZF-DFE is to relax the boundaries of the search region in (10) in order to be able to use tools of lattice theory. It is developped in [10] and briefly reminded here. The goal is to perform a lattice reduction on $\mathbf{H}'$. The first step is to replace $\mathcal{U}$ by the whole range of integers Z. The minimization in (10) can then be approximated by:

$$\widehat{c} = \arg \min_{c \in Z^m} \|\mathbf{y}' - \mathbf{H}'\mathbf{c}\|^2 \quad (16)$$

One can then perform a lattice reduction on $\mathbf{H}'$ to get a new generator matrix:

$$\mathbf{B} = \mathbf{H}'\mathbf{U}, \quad (17)$$

with $\mathbf{U}$ a unimodular matrix ($\mathbf{U}Z^m = Z^m$) and $\mathbf{B}$ the reduced matrix. The minimization problem then becomes:

$$\widehat{c} \simeq \mathbf{U} \arg \min_{\mathbf{c}' \in Z^m} \|\mathbf{y}' - \mathbf{B}\mathbf{c}'\|^2. \quad (18)$$

An adaptive method to compute $\mathbf{U}$, using the Lenstra, Lenstra and Lovász (LLL) reduction [9] is exposed in [10], which will be used in our simulations.

IV. Adaptive Preprocessing and Ordering

In this section, we expose the whole preprocessing and ordering stage and give an adaptive method to perform it taking advantage of the temporal correlation of the channel.

Using the notations of the previous sections, the preprocessing and ordering stage runs in three steps and can be described as follows:

1) $\mathbf{B} = \mathbf{H}'\mathbf{U}$, with $\mathbf{U}$ unimodular and $\mathbf{B}$ having almost orthogonal columns with small norms,
2) $\mathbf{G} = \mathbf{BT}$, with $\mathbf{T}$ a permutation of the columns of $\mathbf{B}$ according to their Euclidean norm,
3) $\mathbf{G} = \left[\begin{array}{cc} \mathbf{Q_1} & \mathbf{Q_2} \end{array} \right] \left[\begin{array}{c} \mathbf{R} \\ \mathbf{0} \end{array} \right]$, with $\mathbf{Q_1} and \mathbf{Q_2}$ orthogonal and $\mathbf{R}$ upper triangular.

An efficient adaptive method for the first step (LLL reduction) has been developped in [10]. We now focus on the second step (columns ordering).

In the following, we denote by $\mathbf{B}_{:,j}$ the j^{th} column of $\mathbf{B}$:

$$\mathbf{B} = \left[\begin{array}{cccc} \mathbf{B}_{:,1} & \mathbf{B}_{:,2} & \cdots & \mathbf{B}_{:,m} \end{array} \right]. \tag{19}$$

Step 2) can be performed as follows:

- compute $\mathbf{A} = [A_1, A_2, ..., A_M]$, where $A_k = \|\mathbf{B}_{:,j}\|^2$, $j = 1, 2, ..., m$,
- apply a sorting algorithm on $\mathbf{A}$ to reorder it in an non-decreasing order.

For each permutation in $\mathbf{A}$ in the sorting algorithm, the permutation matrix $\mathbf{T}$ is updated as follows:

$$A_i A_j \Longleftrightarrow \mathbf{T} = \mathbf{T} \times \mathbf{I}_{i,j} \tag{20}$$

where $\mathbf{I}_{i,j}$ is the modified identity matrix of size m where the columns i and j have been permuted.

The choice of the sorting algorithm depends on the size of M. According to [8], the faster algorithm in the case of a small number of elements in the array $\mathbf{A}$ (typically between 2 and 10 elements) is the *insertion algorithm*.

Considering a slow fading channel, i.e. a small value of the maximum Doppler frequency in the MIMO fading channel model, on can argue that the variations of the elements of $\mathbf{H}'$ are small through time, and so are the variations of $\mathbf{B}$. If $\mathbf{H}'_1$ and $\mathbf{H}'_2$ are two consecutive channel realizations, one can write:

$$\mathbf{H}'_2 = \mathbf{H}'_1 + \Delta\mathbf{H}' \tag{21}$$

where $\Delta\mathbf{H}'$ is a matrix with small elements. Then, with $\mathbf{B}_1 = \mathbf{H}'_1\mathbf{U}_1$ and $\mathbf{B}_2 = \mathbf{H}'_2\mathbf{U}_2$, $\mathbf{U}_1, \mathbf{U}_2$ being the corresponding unimodular matrix, one can write:

$$\mathbf{B}_2 = \mathbf{B}_1 + \Delta\mathbf{B} \tag{22}$$

where $\Delta\mathbf{B}$ has small elements. The variations of the Euclidean norms of $\mathbf{B}$ are then also small. Denoting by $\mathbf{T}_1$ the permutation found for $\mathbf{H}'_1$, one can argue that the matrix $\widetilde{\mathbf{B}}_2 = \mathbf{B}_2\mathbf{T}_1$ is almost ordered as desired. Then, performing step 2) on $\widetilde{\mathbf{B}}_2$ instead of $\mathbf{B}_2$ requires less computational complexity.

In order to reduce significantly the complexity, one can omit some column permutations given the following approximation: if $A_j \leq A_i + \epsilon$, with $1 \leq i, j \leq m$, one consider that $A_j \leq A_i$, even if $A_j > A_i$. The parameter ϵ has to be choosen according to the computational complexity constraints and the desired error performance.

V. Simulation results

We present in this section some simulation results. We have compared the symbol error rate (SER) and the complexity for different values of ϵ. In our case, the complexity of the column ordering step can be described in terms of number of insertions in the insertion algorithm. From our simulations, it appears that the permutation does not affect the SER when performing the LLL reduction before a QR decomposition of the channel matrix.

Fig. 1 shows the SER obtained for a 4×4 MIMO system ($M = 4, N = 4$) and with a QPSK constellation for a certain range of SNR values. One curve represents the SER obtained when the LLL reduction is performed before the ZF-DFE step, and the other curves represent the SER obtained with different values of ϵ without performing any LLL reduction.

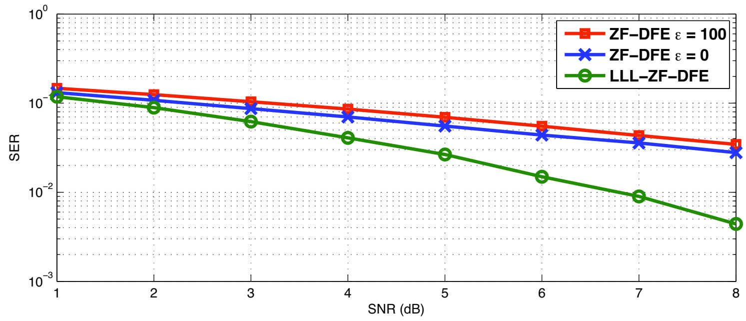

Fig. 1. SER by SNR values. $M = 4, N = 4$, QPSK.

It can be seen that there is a significant gain from a performance point of view when performing the LLL reduction. When no LLL reduction is performed, the loss of performance obtained with the increase of ϵ is is not very significant. Since the loss of error performance is not very high even for great values of ϵ, we have plotted only the cases $\epsilon = 0$ and $\epsilon = 100$. The latter practically corresponds to the case when the permutation is performed once and not updated during the rest of the frame.

We plot in this Fig. 2 the average number of insertions in the sorting algorithm in both the adaptive and non-adaptive cases for over 10^3 channel variations obtained with a QPSK constellation at $SNR = 10$ dB and for different numbers of antennas in square systems ($M = N$).

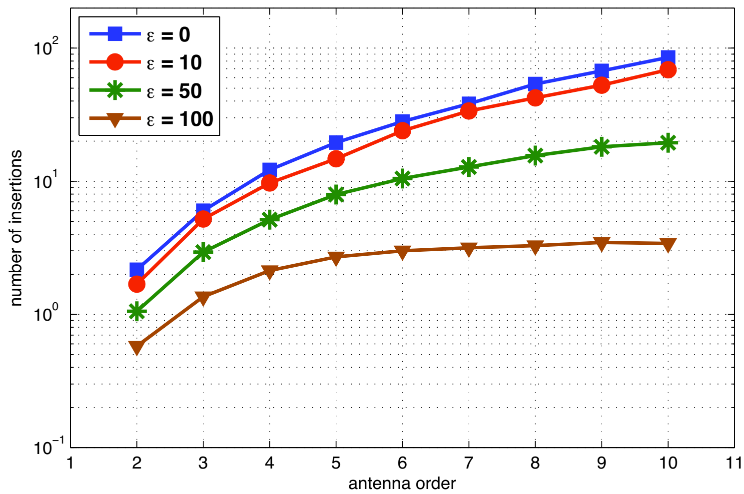

Fig. 2. Number of insertions. $M = N$, QPSK, 10 dB.

In terms of complexity, the gain is very significant, even for small values of ϵ. The value of this parameter is to be choosen according to the complexity constraints and the error peformance one whishes to achieve.

VI. Conclusion

In this work, we have presented an adaptive method for MIMO detection over a slowly varying fading channel. This method takes advantage of the temporal correlation of the channel to allow significant reduction of the complexity of the required preprocessing in this kind of detection. Moreover, the error performance is affected but not very significantly. The value of the adaptive parameter ϵ is to be chosen in an had hoc way according to the error performance one wishes to reach and to the computationnal complexity constraints.

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