Bivariate $\mathcal{G}$ Distribution with Arbitrary Fading Parameters

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***Abstract*—The correlated bivariate $\mathcal{G}$ distribution with arbitrary and not necessarily identical parameters is addressed in this paper. This compound distribution, which is a mixture of arbitrary correlated Rayleigh and inverse Gaussian random variables (RVs), is very convenient for modeling correlated fading shadowing channels. New closed-form expressions for the probability density function (PDF), the cumulative density function (CDF) and the joint moments are provided to statistically characterize the bivariate $\mathcal{G}$ distribution. Furthermore, simpler expressions are obtained when considering independent inverse-Gaussian shadowing. Capitalizing on these theoretical expressions for the statistical characteristics of the correlated $\mathcal{G}$ distribution, the performance analysis of various diversity reception techniques, such as selection diversity (SD) and maximal ratio combining (MRC) over bivariate $\mathcal{G}$ fading channels is presented.**

***Index Terms*—Bit error probability, bivariate $\mathcal{G}$ distribution, correlated multipath-shadowing fading, maximal ratio combining (MRC), outage probability, selection diversity (SD).**

I. Introduction

In recent years, compound statistical modeling of the wireless channel envelope has gained much interest. Several statistical channel models have been proposed for the analysis of communication systems in the presence of the composite propagation environments that appear when multipath fading and shadowing occur simultaneously [1]. Such propagation environments exist in land-mobile satellite systems [2], and in metropolitan areas with slow moving users [1]. The most commonly used distributions for modeling composite propagation environments are the Rayleigh-, Rice-, and Nakagami-lognormal (NL) [1]-[3]. Unfortunately these lognormal-based fading/shadowing channel models, due to their complex mathematical nature, are rather inconvenient for analytically evaluating the performance of digital communication systems in the presence of such fading channels. Recently, many attempts have been made to obtain a practical composite distribution fit in closed-form. For instance, the well known $\mathcal{K}$-distribution [4] and its generalized version [5] proved to be particularly useful in evaluating the performance of composite channels [6]-[7]. Recently, the inverse-Gaussian pdf was proposed as a substitute to the log-normal one. In [6], the authors proved that the compound Rayleigh-inverse Gaussian model, known as the $\mathcal{G}$ distribution, approximates the shadowed Rayleigh fading more accurately than the $\mathcal{K}$-distribution. In [9], capitalizing on the convenient mathematical form of the $\mathcal{G}$ distribution, the outage probability (OP), capacity, and error rates over such fading/shadowing channels have been studied.

In a diversity system, the received diversity signals could be correlated resulting in a degradation of the achievable diversity gain [1]. Typically, such signal correlation exists in relatively small size mobile terminals, where usually the distance between the diversity antennas is small. The open research literature concerning bivariate distributions is quite extensive but with a general observation that previously published papers, dealing with correlated fading channel models, consider only small scale effects [10][14]. In the presence of shadowing, the lognormal statistics and hence the inverse Gaussian statistics become dominant. So far, few works dealing with bivariate distribution in compound channels have been presented. For example in a recent paper [15], the joint PDF of non necessarily identically $\mathcal{K}$-distributed RVs with arbitrary correlation was derived in infinite series expressions. Nevertheless, to the best of the author's knowledge, a detailed analysis of the bivariate $\mathcal{G}$ distribution and the performance of digital communications systems over such composite fading channels, is not available in the open research literature. Motivated by the preceding, in this paper the most important statistical properties of the bivariate $\mathcal{G}$ fading distribution with non identical shaping and scaling parameters are presented and applied to the performance analysis of diversity receivers. The remainder of the paper is organized as follows. In Section II, the PDF and cumulative distribution function (CDF) are derived in infinite series representation, while a closed-form expression for the joint moments is presented. In Section III, the CDF of SD and the moments generating function (MGF) of MRC operating over correlated $\mathcal{G}$ fading channels are derived. These expressions are used to obtain several performance evaluation results presented in Section IV. Finally, we conclude the paper while summarizing the main results in section V.

II. Statistical Characterization of the Correlated $\mathcal{G}$ Distribution

Let us consider two correlated Rayleigh distributed RVs $Y_i, i = 1, 2$, with the joint pdf given by [1]

$$f_{Y_1,Y_2}(y_1, y_2) = \sum_{t=0}^{\infty} \frac{4\rho_R^t}{(t!)^2(1-\rho_R)^{2t+1}} \prod_{i=1}^{2} \frac{y_i^{2t+1}}{W_i^{t+1}} e^{-\frac{y_i^2}{(1-\rho_r)W_i}}, \quad (1)$$

where ρ_R is the power correlation coefficient between Y_1^2 and Y_2^2 and W_i is the fading average power defined as

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$W_i = E_{Y_i}(y_i^2)$, with $E_{Y_i}(\cdot)$ denoting averaging over the distribution of Y_i. When multi-path fading is superimposed on shadowing, typically the scenario in congested downtown areas with a high number of slow moving pedestrian and vehicles, W_i randomly varies. In the following analysis, W_i will be modeled with the inverse Gaussian (IG) distribution [6]. When Shadowing is correlated, W_1 and W_2 are governed by a bivariate inverse Gaussian distribution. Various definitions of bivariate IG-distribution exist in the literature. A "natural" one is given by Kocherlakota (1986), who dismisses a previous suggestion by Al-Hussain and Abd-El-Hakim (1981), and is given by [16]

$$f_{W_1,W_2}(w_1,w_2) = \frac{1}{4\pi}\sqrt{\frac{\lambda_1\lambda_2}{(1-\rho^2)w_1^3 w_2^3}} \left\{ e^{-\frac{1}{2(1-\rho^2)}\left[\frac{\lambda_1(w_1-\mu_1)^2}{\mu_1^2 w_1} - \frac{2\rho}{\mu_1\mu_2}\sqrt{\frac{\lambda_1\lambda_2}{w_1 w_2}}(w_1-\mu_1)(w_2-\mu_2) + \frac{\lambda_2(w_2-\mu_2)^2}{\mu_2^2 w_2}\right]} + e^{-\frac{1}{2(1-\rho^2)}\left[\frac{\lambda_1(w_1-\mu_1)^2}{\mu_1^2 w_1} + \frac{2\rho}{\mu_1\mu_2}\sqrt{\frac{\lambda_1\lambda_2}{w_1 w_2}}(w_1-\mu_1)(w_2-\mu_2) + \frac{\lambda_2(w_2-\mu_2)^2}{\mu_2^2 w_2}\right]} \right\}, \quad (2)$$

where λ_i and μ_i are the shaping parameters and ρ is the correlation coefficient between W_1 and W_2. Many shadowing conditions can be modeled using (2) with different values of λ_i and μ_i. Indeed, in [9], authors have shown that the IG-distribution can be matched to the lognormal distribution by setting

$$\lambda = \frac{e^{\theta}}{2\sinh(\frac{\sigma^2}{2})}, \quad \text{and} \quad \mu = e^{\theta+\frac{\sigma^2}{2}}, \quad (3)$$

where θ and σ are, respectively, the mean and the standard deviation of the lognormal shadowing. The pdf in (2) can be rewritten, after some straight forward manipulations, as

$$f_{W_1,W_2}(w_1,w_2) = \frac{1}{2\pi\sqrt{1-\rho^2}}\cosh\left(\frac{\rho}{1-\rho^2}\frac{(\lambda_1\lambda_2)^{1/2}(w_1-\mu_1)(w_2-\mu_2)}{\mu_1\mu_2\sqrt{w_1 w_2}}\right) \prod_{i=1}^2 \left(\frac{\lambda_i}{w_i^3}\right)^{1/2} e^{-\frac{1}{2(1-\rho^2)}\left[\frac{\lambda_i(w_i-\mu_i)^2}{\mu_i^2 w_i}\right]}. \quad (4)$$

Since W_i's are considered as RVs, (1) is conditioned on W_i's and the total probability theorem may be applied [17, Eq. (7.44)]. Hence, the combined Rayleigh fading and inverse Gaussian shadowing bivariate distribution, also denoted as the $\mathcal{G}$ distribution, can be obtained by averaging (1) with respect to W_i's as

$$f_{R_1,R_2}(x_1,x_2) = \int_0^\infty\int_0^\infty f_{Y_1,Y_2}(x_1,x_2) f_{W_1,W_2}(w_1,w_2)\,dw_1 dw_2. \quad (5)$$

Expanding the $\cosh(\cdot)$ term in (4) in its infinite series form as given in [18], two integrals of the form $I = \int_0^\infty y^\alpha e^{-Ay-\frac{B}{y}}dy$ need to be solved. By the help of [18, Eq. (3.471.9)] and after some manipulations, a closed-form expression of the bivariate $\mathcal{G}$ distribution with different correlation coefficients and not necessarily identical statistical parameters is obtained as

$$f_{R_1,R_2}(x_1,x_2) = \frac{8}{\pi}\sum_{t=0}^{\infty}\frac{\rho_R^t}{(t!)^2(1-\rho_R)^{2t+1}}\sum_{k=0}^{\infty}\frac{\rho^{2k}}{(2k)!(1-\rho^2)^{2k+1/2}} \prod_{i=1}^2 \frac{\lambda_i^{k+1/2}}{\mu_i^{k+t+\frac{3}{2}}} e^{\frac{\lambda_i}{\mu_i(1-\rho^2)}} x_i^{2t+1} \sum_{p=0}^{2k} C_p^{2k}(-1)^{2k-p} \left[d_i x_i^2+1\right]^{\frac{p-k-t-3/2}{2}} K_{p-k-t-3/2}\left[b_i\sqrt{d_i x_i^2+1}\right], \quad (6)$$

where $b_i = \frac{\lambda_i}{\mu_i(1-\rho^2)}$, $d_i = \frac{2(1-\rho^2)}{\lambda_i(1-\rho_R)}$ and $K_\nu(\cdot)$ is the second kind of the modified Bessel function of order ν [18, Eq. (8.407.1)]. A special case is when W_1 and W_2 are uncorrelated, i.e., $\rho = 0$. In this case (6) simplifies to

$$f_{R_1,R_2}(x_1,x_2) = \frac{8}{\pi}\sum_{t=0}^{\infty}\frac{\rho_R^t}{(t!)^2(1-\rho_R)^{2t+1}}\prod_{i=1}^2 \frac{\sqrt{\lambda_i}}{\mu_i^{t+3/2}} e^{\frac{\lambda_i}{\mu_i}} x_i^{2t+1} \frac{K_{t+3/2}\left(b_i\sqrt{d_i x_i^2+1}\right)}{\sqrt{d_i x_i^2+1}^{\,t+3/2}}. \quad (7)$$

By the help of [18, Eq. (6.596.3)], we can easily show that

$$\int_0^\infty\int_0^\infty f_{R_1,R_2}(x_1,x_2)\,dx_1 dx_2 = (1-\rho_R)\sum_{t=1}^{\infty}\rho_R^t = 1. \quad (8)$$

Consequently, (7) is a valid pdf. Moreover, the PDF of the correlated $\mathcal{G}$-distribution with identical parameters can be easily obtained form (6) by assuming identical shaping parameters, i.e., $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$.

The joint CDF of R_1 and R_2 is given by $F_{R_1,R_2}(y_1,y_2) = \int_0^{y_1}\int_0^{y_2} f_{R_1,R_2}(x_1,x_2)dx_1 dx_2$ [17, Eq. (6.6)]. Substituting (6) in this expression and using a change of variable $s = x^2$, yields

$$F_{R_1,R_2}(y_1,y_2) = \frac{8}{\pi}\sum_{t=0}^{\infty}\frac{\rho_R^t}{(t!)^2(1-\rho_R)^{2t+1}}\sum_{k=0}^{\infty}\frac{\rho^{2k}}{(2k)!(1-\rho^2)^{2k+1/2}} \prod_{i=1}^2 \frac{\lambda_i^{4k+1/2}}{\mu_i^{k+t+\frac{3}{2}}} e^{\frac{\lambda_i}{\mu_i(1-\rho^2)}} \sum_{p=0}^{2k} C_p^{2k}(-1)^p \left[I_{k+t-p+1,t+1}(y_i^2,d_i,b_i) - I_{k+t-p+1,t+1}(0,d_i,b_i)\right], \quad (9)$$

where

$$I_{p,q}(x,y,z) = \int x^{q-1}\frac{K_{p+1/2}(z\sqrt{xy+1})}{(\sqrt{xy+1})^{p+1/2}}dx. \quad (10)$$

Using the result of Appendix I, the joint CDF, $F_{R_1,R_2}(y_1,y_2)$, is obtained in closed-form as in (11). In the case of uncorrelated shadowing, the CDF reduces to

$$F_{R_1,R_2}(y_1,y_2) = \frac{2}{\pi}\sum_{t=0}^{\infty}\frac{\rho_R^t}{(1-\rho_R)^{2t+1}}\prod_{i=1}^2 \frac{\sqrt{\lambda_i}}{\mu_i^{t+3/2}} e^{\frac{\lambda_i}{\mu_i}} \left[\mu_i^{t+1}(1-\rho_R)^{t+1}K_{\frac{1}{2}}(b_i) - \sum_{n=1}^{t+1}\frac{\mu_i^n(1-\rho_R)^n}{(t+1-n)!} y_i^{2(t+1-N)} \frac{K_{t+3/2-n}(b_i\sqrt{d_i y_i^2+1})}{(\sqrt{d_i y_i^2+1})^{t+3/2-p}}\right]. \quad (12)$$

The product moment of R_1 and R_2 of order n_1+n_2 is obtained by the help of [18, Eq. (6.596.3)] as

$$E_{R_1,R_2}[x_1^{n_1}x_2^{n_2}] = \frac{2}{\pi}(1-\rho_R)\sum_{t=0}^{\infty}\frac{\rho_R^t}{(t!)^2}\sum_{k=0}^{\infty}\frac{\rho^{2k}}{(2k)!(1-\rho^2)^{2k+1/2}} \prod_{i=1}^2 \frac{\lambda_i^{k+1/2}}{\mu_i^{k-\frac{n_i}{2}+\frac{1}{2}}} e^{\frac{\lambda_i}{\mu_i(1-\rho^2)}}\Gamma\left(\frac{n_i}{2}+t+1\right)(1-\rho_R)^{\frac{n_i}{2}} \sum_{p=0}^{2k} C_p^{2k}(-1)^{2k-p}K_{k-p-\frac{n_i}{2}+\frac{1}{2}}(b_i). \quad (13)$$

$$F_{R_1,R_2}(y_1,y_2) = \frac{2}{\pi}\sum_{t=0}^{\infty}\frac{\rho_R^t}{(1-\rho_R)^{2t+1}}\sum_{k=0}^{\infty}\frac{\rho^{2k}}{(2k)!(1-\rho^2)^{2k+1/2}}\prod_{i=1}^{2}\frac{\lambda_i^{k+1/2}}{\mu_i^{k+t+\frac{3}{2}}}e^{\frac{\lambda_i}{\mu_i(1-\rho^2)}}\sum_{p=0}^{2k}C_p^{2k}(-1)^{2k-p}\left[\mu_i^{t+1}(1-\rho_R)^{t+1}\right.$$
$$\left.K_{p-k+\frac{1}{2}}(b_i)-\sum_{n=1}^{t+1}\frac{\mu_i^n(1-\rho_R)^n y_i^{2(t+1-n)}}{(t+1-n)!}\frac{K_{p-k-t-3/2+n}(d_i\sqrt{b_iy_i^2+1})}{(\sqrt{b_iy_i^2+1})^{p-k-t-3/2+n}}\right]. \tag{11}$$

III. Dual Branch Diversity Receivers Statistics

The previously derived formulas can be very useful to obtain the performance of a wide range of dual-branch diversity receivers operating over correlated composite fading shadowing channels governed by the $\mathcal{G}$ distribution. In a dual branch diversity receiver, the equivalent baseband received signal at the i-th ($i = 1, 2$) antenna is expressed as $x_i = sh_i + n_i$ where s is the transmitted symbol with energy E_s, n_i is the complex additive white Gaussian noise (AWGN) with single sided power spectral density N_0 assumed identical to both branches, and h_i is the channel complex gain such as $R_i = |h_i|$. By assuming uncorrelated noise, the instantaneous signal-to-noise ratio (SNR) per symbol at the i-th input branch γ_i, and the corresponding average SNR, $\bar{\gamma_i}$, can be expressed as

$$\gamma_i = \frac{Es}{N_0}R_i^2, \quad i = 1, 2, \tag{14}$$

and

$$\bar{\gamma_i} = \frac{Es}{N_0}E\{R_i^2\} = \mu_i\frac{Es}{N_0}, \tag{15}$$

respectively. In the following subsections, important statistical metrics for the diversity receivers under consideration, namely SD and MRC will be presented. With the change of variable $\gamma_i = \frac{\bar{\gamma_i}}{\mu_i}y_i^2$ in (6), the pdf of the bivariate instantaneous signal-to-noise power ratio (SNR) $f_{\gamma_1,\gamma_2}(y_1, y_2)$, can be easily deduced as

$$f_{\gamma_1,\gamma_2}(y_1,y_2)=\frac{2}{\pi}\sum_{t=0}^{\infty}\frac{\rho_R^t}{(t!)^2(1-\rho_R)^{2t+1}}\sum_{k=0}^{\infty}\frac{\rho^{2k}}{(2k)!(1-\rho^2)^{2k+1/2}}$$
$$\prod_{i=1}^{2}\left(\frac{\lambda_i}{\mu_i}\right)^{k+\frac{1}{2}}e^{\frac{\lambda_i}{\mu_i(1-\rho^2)}}\frac{\gamma_i^t}{\bar{\gamma_i}^{t+1}}\sum_{p=0}^{2k}C_p^{2k}(-1)^{2k-p}$$
$$[c_i\gamma_i+1]^{\frac{p-k-t-3/2}{2}}K_{p-k-t-3/2}\left[b_i\sqrt{c_i\gamma_i+1}\right], \tag{16}$$

where $c_i = \frac{d_i\mu_i}{\bar{\gamma_i}}$.

A. Selection Combining (SD)

Since the instantaneous SNR at the output of the SD receiver is

$$\gamma_{SD} = max(\gamma_1, \gamma_2), \tag{17}$$

its CDF can be expressed as $F_{\gamma_{SD}}(\gamma) = F_{\gamma_1,\gamma_2}(\gamma, \gamma)$ [17, Eq. (6.54)]. Using this equation and making a change of variable $\gamma_i = \frac{\bar{\gamma_i}}{\mu_i}y_i^2$ yields $F_{\gamma_{SD}}$ given by (18). The OP is defined as the probability that the SD output SNR falls below a predetermined outage threshold γ_{th}. By employing (16), the OP of dual-branch SD can be obtained by replacing γ with γ_{th} in (18) as $Pout = F_{\gamma_{SD}}(\gamma_{th})$.

B. Maximal Ratio Combining (MRC)

1) Moments of the Output SNR: The instantaneous output SNR per symbol of MRC receivers can be expressed as

$$\gamma_{out} = \frac{E_s(R_1^2 + R_2^2)}{N_0}. \tag{19}$$

The n-th moment of γ_{out} denoted by $\mu_n = E\{\gamma_{out}^n\}$ yields

$$\mu_n = \left(\frac{E_s}{N_0}\right)^n E\{(R_1^2 + R_2^2)^n\}. \tag{20}$$

Hence, using the binomial identity and making a change of variables in (13) and after some straight-forward mathematical manipulations, μ_n at the output of the MRC diversity receivers can be obtained in closed form as

$$\mu_n = \frac{2}{\pi}(1-\rho_R)^n\sum_{t=0}^{\infty}\frac{\rho_R^t}{(t!)^2}\sum_{k=0}^{\infty}\frac{\rho^{2k}}{(2k)!(1-\rho^2)^{2k+1/2}}$$
$$\left(\prod_{i=1}^{2}(\frac{\lambda_i}{\mu_i})^{k+\frac{1}{2}}e^{\frac{\lambda_i}{\mu_i(1-\rho^2)}}\right)\sum_{m=0}^{n}C_m^n\Gamma(m+t+1)\bar{\gamma_1}^m\bar{\gamma_2}^{n-m}$$
$$\Gamma(n-m+t+1)\sum_{p_1,p_2=0}^{2k}C_{p_1}^{2k}C_{p_2}^{2k}(-1)^{p_1+p_2}$$
$$K_{k-p_1-m+\frac{1}{2}}(b_1)K_{k-p_2-n+m+\frac{1}{2}}(b_2). \tag{21}$$

From (21), a closed-form expression of the amount of fading (AF), which is a unified measure of the fading severity of a specified channel model [1], can be obtained for the MRC receivers as $AF = \mu_2/\mu_1^2 - 1$.

2) Average Error rate: For an MRC receiver, the instantaneous SNR is $\gamma_{MRC} = \gamma_1 + \gamma_2$. The MGF of γ_{MRC} is therefore defined as $M_{\gamma_{MRC}}(s) = M_{\gamma_1,\gamma_2}(s, s)$, where

$$M_{\gamma_1,\gamma_2}(s,s) = \int_0^{\infty}\int_0^{\infty}e^{-s(y_1+y_2)}f_{\gamma_1,\gamma_2}(y_1,y_2)dy_1dy_2. \tag{22}$$

Substituting (16) in (22), an integral of the form

$$J = \int_0^{\infty}e^{-sy}y^t\frac{K_{k+t+\frac{3}{2}-p}(b\sqrt{cy+1})}{(\sqrt{cy+1})^{k+t+\frac{3}{2}-p}}dy, \tag{23}$$

needs to be solved. In Appendix II, we prove that J can be written in closed-form as

$$J = (-1)^t\left[\sum_{n=t+1}^{v}(-1)^{n-1}2^n\frac{(n-1)!}{(n-1-t)!}\frac{K_{v-n+\frac{1}{2}}(b)}{(bc)^n}s^{n-1-t}\right.$$
$$+(-1)^v\frac{2^v}{(bc)^v}\sum_{q=0}^{t}C_q^t\frac{v!}{(v-t+q)!}s^{v-t+q}\frac{(-1)^q}{d^{q+1}}\sqrt{\frac{\pi}{2b}}e^{-b}\sum_{m=0}^{q}C_m^q$$
$$\left.2^{q-m}(\frac{s}{c})^{-\frac{q+m+1}{2}}\Gamma(q+m+1)H_{-(q+m+1)}(\frac{b}{2}\sqrt{\frac{c}{s}}+\sqrt{\frac{s}{c}})\right], \tag{24}$$

where $v = k + t + 1 - p$ and $H_\nu(x)$ is the Hermite function of order ν given in [19] as

$$H_\nu(x) = 2^\nu\sqrt{\pi}\left[\frac{{}_1F_1(-\frac{\nu}{2},\frac{1}{2},x^2)}{\Gamma(\frac{1-\nu}{2})} - \frac{2x{}_1F_1(\frac{1-\nu}{2},\frac{3}{2},x^2)}{\Gamma(-\frac{\nu}{2})}\right], \tag{25}$$

$$F_{\gamma_{SD}}(\gamma) = \frac{2}{\pi}\sum_{t=0}^{\infty} \frac{\rho_R^t}{(1-\rho_R)^{2t+1}} \sum_{k=0}^{\infty} \frac{\rho^{2k}}{(2k)!(1-\rho^2)^{2k+1/2}} \prod_{i=1}^{2} (\frac{\lambda_i}{\mu_i})^{k+1/2} e^{\frac{\lambda_i}{\mu_i(1-\rho^2)}} \sum_{p=0}^{2k} C_p^{2k}(-1)^p \Big[(1-\rho_R)^{t+1} K_{p-k+\frac{1}{2}}(b_i) - \sum_{n=1}^{t+1} \frac{(1-\rho_R)^n \left(\frac{\gamma}{\bar{\gamma}_i}\right)^{t+1-n}}{(t+1-n)!} \frac{K_{p-k-t-3/2+n}(b_i\sqrt{c_i\gamma_i+1})}{(\sqrt{c_i\gamma_i+1})^{p-k-t-3/2+n}}\Big]. \tag{18}$$

$$M_{\gamma_{MRC}}(s) = \frac{4}{\pi} \sum_{t=0}^{\infty} \frac{\rho_R^t}{(t!)^2(1-\rho_R)^{2t+1}} \sum_{k=0}^{\infty} \frac{\rho^{2k}}{(2k)!(1-\rho^2)^{2k+1/2}} \prod_{i=1}^{2} \left(\frac{\lambda_i}{\mu_i}\right)^{k+\frac{1}{2}} e^{\frac{\lambda_i}{\mu_i(1-\rho^2)}} \frac{1}{\bar{\gamma}_i{}^{t+1}} \sum_{p=0}^{2k} C_p^{2k}(-1)^{2k-p} \Big[\sum_{n=t+1}^{v}(-1)^{n-1}(1-\rho_R)^n\bar{\gamma}_i{}^n \frac{(n-1)!}{(n-1-t)!} K_{v-n+\frac{1}{2}}(b_i)s^{n-1-t} + (-1)^v(1-\rho_R)^v\bar{\gamma}_i{}^v \sum_{q=0}^{t} C_q^t \frac{v!}{(v-t+q)!} s^{k-p} (-1)^q \sqrt{\frac{2\pi}{b_i}} e^{-b_i} \sum_{m=0}^{q} C_m^q 2^{q-m} (\frac{s}{c_i})^{\frac{q+1-m}{2}} \Gamma(q+m+1) H_{-(q+m+1)}(\frac{b_i}{2}\sqrt{\frac{c_i}{s}} + \sqrt{\frac{s}{c_i}})\Big] \tag{26}$$

where ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent Hypergeometric function of the first kind [19]. After some manipulations, the MGF of γ_{MRC} can be expressed as in (26). When the two branches undergo uncorrelated shadowing, (26) reduces to

$$M_{\gamma_{MRC}}(s) = 2(1-\rho_R)\sum_{t=0}^{\infty} \rho_R^t \prod_{i=1}^{2}\Big[1-(t+1)\sum_{m=0}^{t}(-1)^m \frac{C_m^t}{(m+1)!} \sum_{n=0}^{m} C_n^m (2\sqrt{\frac{s}{\alpha_i}})^{m+1-n} \Gamma(m+n+1) H_{-(n+m+1)}(\frac{\beta_i}{2}\sqrt{\frac{\alpha_i}{s}} + \sqrt{\frac{s}{\alpha_i}})\Big], \tag{27}$$

where $\alpha_i = \frac{2\mu_i}{\lambda_i\bar{\gamma}_i(1-\rho_R)}$ and $\beta_i = \frac{\lambda_i}{\mu_i}$.
Having the MGF of γ_{MRC} in closed form as in (26), and using the MGF-based approach for the performance evaluation of digital modulations over fading channels [1], we can obtain the average symbol error rate for a wide range of M-ary modulations. For example, the average SER (Symbol Error Rate) for M-ary phase sift keying M-PSK can be written as [1]

$$P_e = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} M_{\gamma_{MRC}}\left(\frac{\rho}{\sin^2(\phi)}\right) d\phi, \tag{28}$$

where $\rho = \sin^2(\frac{\pi}{M})$. For Binary Frequency Shift Keying modulation (FSK), the bit error rate is obtained by setting $M = 2$ in (32), which can be approximated as follows using [20]

$$P_e \cong \frac{1}{12}M_{\gamma_{MRC}}(\frac{1}{2}) + \frac{1}{4}M_{\gamma_{MRC}}(\frac{4}{6}), \tag{29}$$

which in turn can be directly computed form the MGF without an additional numerical integration. For non-coherent BFSK (NCBFSK) and differential BPSK (DBPSK) the bit error rate is given by

$$P_e = 0.5M_{\gamma_{MRC}}(-b), \tag{30}$$

with $b = 1$ for DBPSK and $b = 0.5$ for NCBFSK. Note that most of the performance results concerning wireless multi-hop transmissions are given for DBPSK.

IV. Numerical Results

Using the previous presented analysis, the OP performance of a dual-branch SD receiver operating over correlated fading channels has been numerically evaluated. In Fig. 1, the OP performance is plotted as a function of the normalized outage threshold where we assume equal average SNR on the two branches, i.e., $\bar{\gamma_1} = \bar{\gamma_2} = \bar{\gamma}$ for different shadowing scenarios and values of ρ_R. As expected, the obtained performance results show that as $\gamma_{th}/\bar{\gamma}$ decreases, the outage performance improves, while the performance also improves as ρ_R decreases and/or as the shadowing becomes more pronounced.

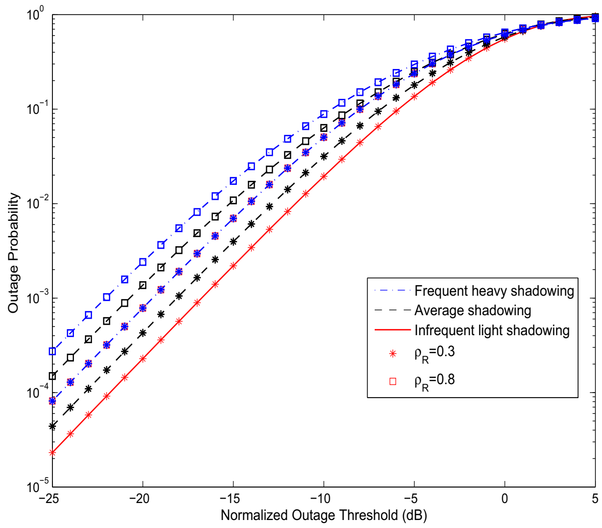

Fig. 1. Outage probability at the output of a dual-branch SD receiver versus the normalized outage threshold, $\gamma_{th}/\bar{\gamma}$, in infrequent light shadowing ($\theta = 0.115$ and $\sigma = 0.115$), average shadowing ($\theta = -0.115$ and $\sigma = 0.161$) and frequent heavy shadowing ($\theta = -3.914$ and $\sigma = 0.161$) environments.

In Fig. 2, the ABEP of DBPSK, FSK and noncoherent FSK signals with MRC diversity receivers is plotted as a function of average input SNR. These receivers operate in i.i.d $\mathcal{G}$ fading channels with $\rho_R = \rho = 0.5$. The ABEP performance clearly improves when the correlations decrease.

V. Conclusion

In this paper, the arbitrary correlated $\mathcal{G}$ distribution with arbitrary and not necessarily identical parameters was introduced and studied. Closed-form expressions of several statistical metrics, namely the PDF, CDF and joint moments have been presented. The obtained formulas have been used to evaluate the performance of dual-branch selection and MRC diversity receivers operating over bivariate $\mathcal{G}$ fading channels.

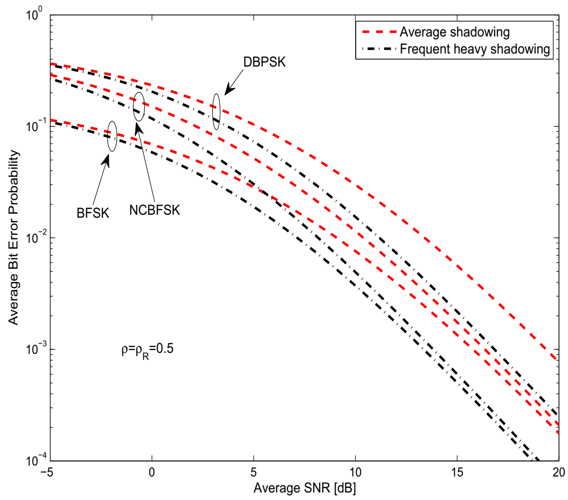

Fig. 2. MRC performances for DPSK, FSK and noncoherent FSK signals versus $\bar{\gamma}$ in average shadowing ($\theta = -0.115$ and $\sigma = 0.161$) and frequent heavy shadowing ($\theta = -3.914$ and $\sigma = 0.161$) environments.

Appendix I: Evaluation of $I_{p,q}(x, y, z)$

Using the fact that [18, Eq. (8.486.15)]

$$\frac{dx^{-s}K_s(x)}{dx} = -x^{-s}K_{s+1}(x), \tag{31}$$

we integrate by parts (10) to obtain

$$I_{p,q}(x, y, z) = -(q-1)!\sum_{k=0}^{q}\frac{2^k x^{q-k}}{(yz)^k(q-k)!}\frac{K_{p-k+\frac{1}{2}}(z\sqrt{xy+1})}{(\sqrt{xy+1})^{p-k+\frac{1}{2}}}. \tag{32}$$

Appendix I: Evaluation of J

We have

$$J = (-1)^t\frac{d^t}{ds^t}\underbrace{\int_0^\infty e^{-sy}\frac{K_{v+\frac{1}{2}}(b\sqrt{cy+1})}{(\sqrt{cy+1})^{v+\frac{1}{2}}}dy}_{f_\nu(s)}, \tag{33}$$

where $\nu = k+t+1-p$. Premeditating an integration by part, we find that

$$f_\nu(s) = \frac{2}{bc}K_{v+\frac{1}{2}}(b) - \frac{2s}{bc}f_{\nu-1}(s). \tag{34}$$

By iterating on the above equation, we find that

$$f_\nu(s) = \sum_{n=1}^{v}(-1)^{n-1}\frac{2^n s^{n-1}}{(bc)^n}K_{v+\frac{1}{2}-n}(b)+(-1)^v\frac{2^v}{(bc)^v}s^v f_0(s), \tag{35}$$

where $f_0(s) = \sqrt{\frac{\pi}{2b}}\int_0^\infty e^{-sy}\frac{e^{-b\sqrt{cy+1}}}{\sqrt{cy+1}}dy$. By inserting (35) into (33), we obtain

$$\begin{aligned}J = (-1)^t\textstyle\sum_{n=t+1}^{v}\frac{(-1)^{n-1}2^n}{(bc)^n}s^{n-1-t}\frac{(n-1)!}{(n-1-t)!}K_{v+\frac{1}{2}-n}(b)\\ +(-1)^v\frac{2^v}{(bc)^v}\textstyle\sum_{q=0}^{t}C_q^t\frac{(k+t+1-p)!}{(k+1-p+q)!}s^{k+1-p+q}f_0(s)^{(q)}.\end{aligned} \tag{36}$$

The q-th derivative of $f_0(t)$ is given after some manipulations, using [18, Eq. (3.462.1)], as

$$\begin{aligned}f_0^{(q)}(s) = \tfrac{(-1)^q}{c^{q+1}}\sqrt{\tfrac{2\pi}{b}}e^{-b}\textstyle\sum_{m=0}^{q}C_m^q 2^{q-m}\left(\frac{s}{c}\right)^{-\frac{(q+m+1)}{2}}\\ \Gamma(q+m+1)H_{-(q+m+1)}\left(\tfrac{b}{2}\sqrt{\tfrac{c}{s}}+\sqrt{\tfrac{s}{c}}\right).\end{aligned} \tag{37}$$

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