ON OPTIMAL BEAMFORMING FOR NOISE REDUCTION AND INTERFERENCE REJECTION

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ABSTRACT

In this paper, we study the performance of the minimum variance distortionless response (MVDR) and linearly constrained minimum variance (LCMV) noise reduction filters when a source of interference and ambient noise coexist with the target signal. We demonstrate that both filters are related as we decompose the MVDR filter into the LCMV and a matched filter (MVDR solution in the absence of interference). Both components are properly weighted to achieve maximum interference-plus-noise reduction at each frequency bin. Furthermore, we elaborate new closed-form expressions for the signal-to-interference ratio (SIR) and signal-to-noise ratio (SNR) at the output of the LCMV, MVDR, and matched filters. These expressions theoretically prove that a tradeoff between noise reduction and interference rejection has to be made. In fact, the total removal of the interference may severely amplify the output ambient noise. Conversely, totally focussing on noise reduction leads to increased level of residual interference. The proposed study is finally supported by numerical examples.

Index Terms— Noise reduction, speech enhancement, minimum variance distortionless response (MVDR), linearly constrained minimum variance (LCMV).

1. INTRODUCTION

In real-world environments, the signals captured by a set of microphones in a speech communication system are mixtures of the desired signal, interference, and ambient noise. A promising solution for proper speech acquisition (with reduced noise and interference) in this context consists in using the LCMV beamformer to reject the interference, reduce the ambient noise energy, and preserve the target signal. The MVDR is also commonly known to reduce the interference-plus-noise energy without distorting the desired signal. In either case, it is of paramount importance to accurately quantify the achieved noise and interference reduction. Indeed, it is quite reasonable to ask, for instance, about the price that has to be paid in order to achieve total removal of the interference without distorting the target signal when using the LCMV. Besides, it is fundamental to understand the effect of the MVDR on both noise and interference.

So far, a substantial progress has been made in the noise reduction literature after the pioneering work of Schroeder in 1965 [1]. The utilization of microphone arrays allows for more flexibility than single-channel processing since more noise reduction can be achieved with theoretically low or even no speech distortion [2, 3, 4]. These gains result from the extra degrees of freedom pertaining to the spatial aperture of the microphone array. Besides the great efforts to develop reliable noise reduction techniques [3, 5, 6, 7, 8, 9], many contributions have been proposed to understand their functioning and accurately quantify their gains and losses in terms of speech distortion and noise reduction. In [10], Bitzer et al. investigated the theoretical performance limits of the generalized sidelobe canceller (GSC) in the case of a spatially diffuse noise. In [11], the theoretical equivalence between the LCMV and its GSC counterpart was demonstrated. In [12], Spriet et al. analyzed the robustness of the Griffiths and Jim GSC [8] and multichannel Wiener filters for hearing aids applications. It was found that the multichannel Wiener filter is not affected by microphones calibration in contrast to the GSC. Recently, Gannot and Cohen have studied the noise reduction ability of the channel transfer function (TF) ratios-based GSC beamformer in [13]. They found that it is theoretically possible to achieve infinite noise reduction when only a coherent noise is added to the speech. In [4], analytical results showing the tradeoff between noise reduction and speech distortion in the parameterized multichannel Wiener filtering were established.

In this paper, we assume that both interference and ambient noise (e.g., spatially white and/or diffuse) coexist with the target source. This assumption is plausible when hands-free full duplex communication devices are deployed within a teleconferencing room, for instance. In this situation, the target signal is generated by one speaker while the interference is more likely to be generated by another participant, or a device (e.g., fan or computer) located within the same room. In addition, ambient noise is ubiquitous in these environments and it is quite reasonable to take it into consideration. A clear understanding of the functioning of noise reduction algorithms in terms of both interference and ambient noise reduction in this case is crucial. Here, we are interested in reducing the noise and interference without distorting the target signal. Therefore, we study both the LCMV and MVDR beamformers (described above) in terms of output SIR and SNR and theoretically establish the tradeoff of interference rejection versus ambient noise reduction.

2. SIGNAL PROPAGATION MODEL AND DEFINITIONS

We consider the following frequency domain representation of the investigated data model [6]

$$Y_n(j\omega) = G_n(j\omega)S(j\omega) + D_n(j\omega)\Psi(j\omega) + V_n(j\omega), \qquad (1)$$

where $Y_n(j\omega)$, $G_n(j\omega)$, $S(j\omega)$, $D_n(j\omega)$, $\Psi(j\omega)$, and $V_n(j\omega)$ are discrete-time Fourier transforms (DTFT's) of the output of microphone n ($n = 1, ..., N$), the channel TF between the target source and microphone n, the channel TF between the interfering source and microphone n, the target speech signal, the original interference signal, and the ambient noise component seen by microphone n, respectively. Also, we define $X_n(j\omega) = G_n(j\omega)S(j\omega)$ and $I_n(j\omega) = D_n(j\omega)\Psi(j\omega)$.

Our aim is to recover one of the noise-free speech components, say $X_1(j\omega)$, the best way we can (along some criteria to be defined later) by applying a linear filter $\mathbf{h}(j\omega)$ to the overall observation vector $\mathbf{y}(j\omega) = [Y_1(j\omega)\ Y_2(j\omega)\ \cdots\ Y_N(j\omega)]^T$. $(\cdot)^T$ denotes the transpose operator. The output of $\mathbf{h}(j\omega)$ is given by

$$\begin{aligned} Z(j\omega) &= \mathbf{h}^H(j\omega)\mathbf{y}(j\omega) \\ &= \mathbf{h}^H(j\omega)\mathbf{x}(j\omega) + \mathbf{h}^H(j\omega)\mathbf{i}(j\omega) + \mathbf{h}^H(j\omega)\mathbf{v}(j\omega), \end{aligned} \quad (2)$$

where $\mathbf{x}(j\omega)$, $\mathbf{i}(j\omega)$, and $\mathbf{v}(j\omega)$ are defined as $\mathbf{y}(j\omega)$, $\mathbf{h}^H(j\omega)\mathbf{x}(j\omega)$ is the filtered speech component, $\mathbf{h}^H(j\omega)\mathbf{i}(j\omega)$ is the residual interference, $\mathbf{h}^H(j\omega)\mathbf{v}(j\omega)$ is the residual noise, and $(\cdot)^H$ is the transpose-conjugate operator. The two vectors containing all the channel TFs between the source, interference, and microphones' locations are $\mathbf{g}(j\omega) = [G_1(j\omega),\ G_2(j\omega),\ ...,\ G_N(j\omega)]^T$ and $\mathbf{d}(j\omega) = [D_1(j\omega),\ D_2(j\omega),\ ...,\ D_N(j\omega)]^T$.

To assess the performance of $\mathbf{h}(j\omega)$, we will use the following two performance measures which are the SNR and SIR at its output. These two measures are respectively defined as

$$\mathrm{SNR_o}\left[\mathbf{h}(j\omega)\right] = \frac{\mathbf{h}^H(j\omega)\mathbf{\Phi}_{xx}(j\omega)\mathbf{h}(j\omega)}{\mathbf{h}^H(j\omega)\mathbf{\Phi}_{vv}(j\omega)\mathbf{h}(j\omega)}, \quad (3)$$

$$\mathrm{SIR_o}\left[\mathbf{h}(j\omega)\right] = \frac{\mathbf{h}^H(j\omega)\mathbf{\Phi}_{xx}(j\omega)\mathbf{h}(j\omega)}{\mathbf{h}^H(j\omega)\mathbf{\Phi}_{ii}(j\omega)\mathbf{h}(j\omega)}, \quad (4)$$

where $\mathbf{\Phi}_{aa}(j\omega) = E\left\{\mathbf{a}(j\omega)\mathbf{a}^H(j\omega)\right\}$ is the power spectrum density (PSD) matrix of a given vector of random processes $\mathbf{a}(j\omega)$. We assume that $\mathbf{\Phi}_{vv}(j\omega)$ is invertible, which is a plausible assumption [3, 13]. The matrices $\mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{\Phi}_{xx}(j\omega)$ and $\mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{\Phi}_{ii}(j\omega)$ are each of rank 1 and their two strictly positive eigenvalues are denoted as $\gamma_{x,v}(\omega) = \mathrm{tr}\left[\mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{\Phi}_{xx}(j\omega)\right]$ and $\gamma_{i,v}(\omega) = \mathrm{tr}\left[\mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{\Phi}_{ii}(j\omega)\right]$, respectively. $\mathrm{tr}\left[\cdot\right]$ denotes the trace of a square matrix. We further define the *collinearity factor*

$$\kappa(\omega) = \frac{\left|\mathbf{g}^H(j\omega)\mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{d}(j\omega)\right|^2}{\mathbf{g}^H(j\omega)\mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{g}(j\omega)\mathbf{d}^H(j\omega)\mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{d}(j\omega)}. \quad (5)$$

Note that $0 \leq \kappa(\omega) \leq 1$. The larger is $\kappa(\omega)$, the more collinear are $\mathbf{\Phi}_{vv}^{-1/2}(j\omega)\mathbf{g}(j\omega)$ and $\mathbf{\Phi}_{vv}^{-1/2}(j\omega)\mathbf{d}(j\omega)$ which are nothing but the propagation vectors of the desired signal and the interference, respectively, up to the linear transformation $\mathbf{\Phi}_{vv}^{-1/2}(j\omega)$. Hence, $\kappa(\omega)$ can be used to characterize the spatial separation between the source and the interference, especially when the noise is spatially white. This factor will be shown to have a direct impact on the performance of the studied filters.

3. FORMULATION OF THE MVDR AND LCMV BEAMFORMERS

The LCMV and MVDR beamformers share the common objectives of attempting to reduce the noise and interference while preserving the target signal. To meet the second objective of both filters, we impose the following constraint [2, 3, 4, 13]

$$\mathbf{g}^H(j\omega)\mathbf{h}(j\omega) = G_1^*(j\omega). \quad (6)$$

This constraint will be taken into consideration in the formulations of both filters.

3.1. Minimum Variance Distortionless Response Beamformer

In the general formulation of the MVDR for noise reduction, the recovery of the noise-free signal consists in minimizing the overall interference-plus-noise energy subject to a no speech distortion constraint. Then, the MVDR beamformer is mathematically obtained by solving the following optimization problem

$$\begin{aligned} \mathbf{h}_{\mathrm{MVDR}}(j\omega) &= \arg\min_{\mathbf{h}(j\omega)} \mathbf{h}^H(j\omega)\left[\mathbf{\Phi}_{ii}(j\omega) + \mathbf{\Phi}_{vv}(j\omega)\right]\mathbf{h}(j\omega) \\ \text{subject to} \quad & \mathbf{g}^H(j\omega)\mathbf{h}(j\omega) = G_1^*(j\omega). \end{aligned} \quad (7)$$

In [5], a first implementation of the MVDR using the GSC structure based on the channel TFs was first proposed for noise and reverberation reduction. Later, the channel transfer function ratios were used to implement the GSC version of the resulting filter in [3]. In [2, 4], a more simplified form that relies on the overall noise and target signal PSD matrices was proposed and is given by

$$\mathbf{h}_{\mathrm{MVDR}}(j\omega) = \frac{\left[\mathbf{\Phi}_{ii}(j\omega) + \mathbf{\Phi}_{vv}(j\omega)\right]^{-1}\mathbf{\Phi}_{xx}(j\omega)\mathbf{u}_1}{\mathrm{tr}\left\{\left[\mathbf{\Phi}_{ii}(j\omega) + \mathbf{\Phi}_{vv}(j\omega)\right]^{-1}\mathbf{\Phi}_{xx}(j\omega)\right\}}. \quad (8)$$

When only the ambient noise $\mathbf{v}(j\omega)$ is superimposed to the desired signal [i.e., $\mathbf{i}(j\omega) = \mathbf{0}$], the MVDR solution becomes

$$\mathbf{h}_{\mathrm{MATCH}}(j\omega) = \frac{\mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{\Phi}_{xx}(j\omega)\mathbf{u}_1}{\gamma_{x,v}(\omega)}. \quad (9)$$

In the sequel, $\mathbf{h}_{\mathrm{MATCH}}(j\omega)$ is termed as *matched filter*.

3.2. Linearly Constrained Minimum Variance Beamformer

To remove the interference through spatial filtering, a common practice has been to zero the array response toward its direction of arrival. In our case, however, we consider the general channel TFs between the interference location and each of the microphone elements. Also, we require the constraint (6) to be satisfied. Thus, we impose the following constraint

$$\mathbf{C}^H(j\omega)\mathbf{h}(j\omega) = \tilde{\mathbf{u}}_1, \quad (10)$$

where $\mathbf{C}(j\omega) = [\mathbf{g}(j\omega)\ \mathbf{d}(j\omega)]$ and $\tilde{\mathbf{u}}_1 = [G_1^*(j\omega)\ 0]^T$. The ambient noise has no specific structure. Therefore, the best that we can do to alleviate its effect is by reducing its energy at the output of $\mathbf{h}(j\omega)$. Then, the optimization problem that zeros the interference, reduces the noise, and preserves the speech is

$$\begin{aligned} \mathbf{h}_{\mathrm{LCMV}}(j\omega) &= \arg\min_{\mathbf{h}(j\omega)} \mathbf{h}^H(j\omega)\mathbf{\Phi}_{vv}(j\omega)\mathbf{h}(j\omega) \\ \text{subject to} \quad & \mathbf{C}^H(j\omega)\mathbf{h}(j\omega) = \tilde{\mathbf{u}}_1. \end{aligned} \quad (11)$$

The solution to (11) is given by:

$$\mathbf{h}_{\mathrm{LCMV}}(j\omega) = \mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{C}(j\omega)\left[\mathbf{C}^H(j\omega)\mathbf{\Phi}_{vv}^{-1}(j\omega)\mathbf{C}(j\omega)\right]^{-1}\tilde{\mathbf{u}}_1. \quad (12)$$

3.3. Relationship Between the MVDR and LCMV

In [6, 13], it was observed that when only spatially coherent noise (termed interference herein) overlaps with the desired source, the GSC (equivalently its MVDR counterpart) is able to totally remove it. This fact does not seem to be straightforward to observe in the general expression of the MVDR filter since a fundamental requirement for this beamformer to exist is that the noise PSD matrix is invertible. To overcome this issue, Gannot and Cohen resorted to regularizing this matrix with a very small factor [13]. Then, it was observed that when this regularization factor is negligible, the MVDR steers a zero toward the interference. This behavior actually reminds us of the LCMV beamformer which passes the desired signal through and rejects the interference. Intuitively, a relationship between both beamformers seems to exist in general situations where both interference and ambient noise coexist. Herein, we confirm this intuition and establish of a new simplified relationship between both filters. First, we apply the matrix inversion lemma to $[\boldsymbol{\Phi}_{ii}(j\omega) + \boldsymbol{\Phi}_{vv}(j\omega)]$ to obtain the following equivalent expression for the MVDR

$$\mathbf{h}_{\text{MVDR}}(j\omega) = \frac{\mathbf{M}_1(j\omega)\boldsymbol{\Phi}_{vv}^{-1}(j\omega)\boldsymbol{\Phi}_{xx}(j\omega)\mathbf{u}_1}{\gamma(\omega) + \gamma_{x,v}(\omega)}, \quad (13)$$

where $\mathbf{M}_1(j\omega) = [1 + \gamma_{i,v}(\omega)]\,\mathbf{I} - \boldsymbol{\Phi}_{vv}^{-1}(j\omega)\boldsymbol{\Phi}_{ii}(j\omega)$, $\gamma(\omega) = \gamma_{i,v}(\omega)\gamma_{x,v}(\omega)\,[1 - \kappa(\omega)]$. We can also demonstrate from (12) that we have

$$\mathbf{h}_{\text{LCMV}}(j\omega) = \frac{\mathbf{M}_2(j\omega)\boldsymbol{\Phi}_{vv}^{-1}(j\omega)\boldsymbol{\Phi}_{xx}(j\omega)\mathbf{u}_1}{\gamma(\omega)}, \quad (14)$$

where $\mathbf{M}_2(j\omega) = \gamma_{i,v}(\omega)\mathbf{I} - \boldsymbol{\Phi}_{vv}^{-1}(j\omega)\boldsymbol{\Phi}_{ii}(j\omega)$. Using (13) and (14), we find that

$$\mathbf{h}_{\text{MVDR}}(j\omega) = \rho_1(\omega)\mathbf{h}_{\text{LCMV}}(j\omega) + [1 - \rho_1(\omega)]\,\mathbf{h}_{\text{MATCH}}(j\omega), \quad (15)$$

where $\rho_1(\omega) = \frac{\gamma(\omega)}{\gamma(\omega)+\gamma_{x,v}(\omega)}$. Clearly, $0 \le \rho_1(\omega) \le 1$.

The new relationship (15) has a very attractive form in which we see that the MVDR attempts to both reducing the ambient noise by means of $\mathbf{h}_{\text{MATCH}}(j\omega)$ and rejecting the interference by means of $\mathbf{h}_{\text{LCMV}}(j\omega)$. Both components are properly weighted to prevent the target signal distortion and achieve a tradeoff between both functions.

4. PERFORMANCE ANALYSIS

The behaviors of the MVDR, LCMV, and matched filters are investigated herein in terms of output SNR and SIR. For the sake of generalization, we define a new parameterized beamformer

$$\mathbf{h}_{\text{p}}(j\omega) = \rho(\omega)\mathbf{h}_{\text{LCMV}}(j\omega) + [1 - \rho(\omega)]\,\mathbf{h}_{\text{MATCH}}(j\omega), \quad (16)$$

where $\rho(\omega)$ is a tuning parameter that satisfies the condition $0 \le \rho(\omega) \le 1$ in order to have a distortionless response. Clearly, the LCMV, MVDR, and matched filters are particular cases of (16) with $\rho(\omega)$ being equal to 1, $\rho_1(\omega)$, and 0, respectively.

Using (9), (14), and (16) jointly with (3) and (4), we find that

$$\text{SNR}_{\text{o}}[\mathbf{h}_{\text{p}}(j\omega)] = \gamma_{x,v}(\omega)\frac{1 - \kappa(\omega)}{1 - [1 - \rho^2(\omega)]\,\kappa(\omega)}, \quad (17)$$

$$\text{SIR}_{\text{o}}[\mathbf{h}_{\text{p}}(j\omega)] = \frac{\gamma_{x,v}(\omega)}{\gamma_{i,v}(\omega)} \cdot \frac{1}{[1 - \rho(\omega)]^2\,\kappa(\omega)}. \quad (18)$$

Then, we draw out two important remarks.

Remark 1: by increasing $\rho(\omega)$, the parameterized filter is more focussed on interference reduction. The extreme case $\rho(\omega) = 1$ corresponds to the LCMV which totally removes the interference, while the other extreme $\rho(\omega) = 0$, corresponding to the matched filter, ignores the interference and uniquely focusses on ambient noise reduction. The third extreme case corresponds to the MVDR which attempts to minimize the overall interference-plus-noise. Actually, we can easily prove by using (17) and (18) that $\text{SNR}_{\text{o}}[\mathbf{h}_{\text{p}}(j\omega)]$ and $\text{SIR}_{\text{o}}[\mathbf{h}_{\text{p}}(j\omega)]$ have opposite variations when $\rho(\omega)$ is varied. Indeed, $\text{SIR}_{\text{o}}[\mathbf{h}_{\text{p}}(j\omega)]$ (respectively, $\text{SNR}_{\text{o}}[\mathbf{h}_{\text{p}}(j\omega)]$) increases (respectively, decreases) with respect to $\rho(\omega)$. For the three particular beamformers above, we have: $\text{SNR}_{\text{o}}[\mathbf{h}_{\text{MATCH}}(j\omega)] \ge \text{SNR}_{\text{o}}[\mathbf{h}_{\text{MVDR}}(j\omega)] \ge \text{SNR}_{\text{o}}[\mathbf{h}_{\text{LCMV}}(j\omega)]$ and $\text{SIR}_{\text{o}}[\mathbf{h}_{\text{MATCH}}(j\omega)] \le \text{SIR}_{\text{o}}[\mathbf{h}_{\text{MVDR}}(j\omega)] \le \text{SIR}_{\text{o}}[\mathbf{h}_{\text{LCMV}}(j\omega)]$.

Remark 2: the collinearity factor $\kappa(\omega)$ plays a fundamental role in the performance of these filters. Indeed, for a given $\rho(\omega) \neq 1$, increasing $\kappa(\omega)$ (by physically placing the noise source near the desired speech in the case of a white noise) leads to smaller output SNR and output SIR. The problem becomes quite complicated if we consider a reverberant enclosure where the existence of some frequencies for which $\kappa(\omega)$ has large values is more likely to be encountered than in anechoic environments for given spatial locations of the interference and the target signal. In such frequencies, the ambient noise can be amplified depending on the choice of $\rho(\omega)$. For the LCMV filter, the output interference is always set to 0 at the price of a decreased output SNR that can reach very small values if $\kappa(\omega) \to 1$.

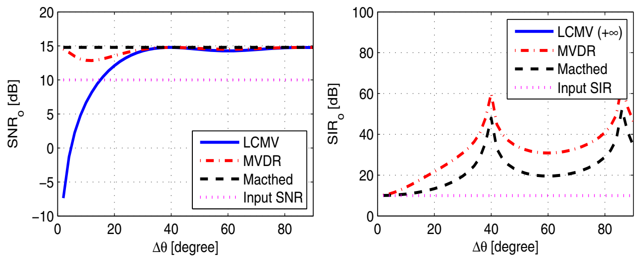
5. NUMERICAL EXAMPLES

For illustration purposes, we consider a uniform linear array (ULA) of three microphones with δ being the inter-microphone spacing, located in a reverberant enclosure simulated using the image method [14]. The microphone elements are placed on the axis $(y_0 = 1.016, z_0 = 1.016)$ m with the center of the array being at $(x_0 = 1.524 \text{ m}, y_0, z_0)$ and the nth microphone at $(x_0 - \frac{N-2n+1}{2}\delta, y_0, z_0)$ with $n = 1, ..., N$. The source and the interference are located in the same enclosure at a distance 2.5 m from the array center and at the azimuthal angles $\theta_s = 120°$ and $\theta_i = \theta_s - \Delta\theta$ which are measured counter-clockwise from the array axis. $\Delta\theta$ will be chosen depending on the examples investigated below. An additive spatially white noise was added to model the ambient noise. Two setups are investigated: anechoic ($T_{60} = 0$ ms) and reverberant ($T_{60} = 200$ ms) rooms. We choose the input SIR and SNR to be equal to 10 dB.

In the first setup ($T_{60} = 0$ ms), we investigate the effect of the angular separation $\Delta\theta$ on the performance of the MVDR, LCMV, and matched filters. The performance of the filters is assessed at a frequency $f = 1$ kHz and the inter-microphones spacing is set such that $\delta = \frac{c}{2f}$ ($c = 343$ ms^{-1} is the speed of sound). Figs. 1 (a) and (b) depict the effect of $\Delta\theta$ on the SIR and SNR at the output of the three beamformers. As $\Delta\theta$ decreases, the output SNR of the LCMV is decreased; the output SNR is even lower than the input SNR for $\Delta\theta < 15°$. The output SNR of the MVDR and matched filters are almost unaffected while very low output SIR values are obtained for small $\Delta\theta$. To better understand this, we provide the normalized beampatterns of the three filters in Fig. 2 for $\Delta\theta = 60°$ and $10°$. Note that when $\Delta\theta$ is small, two major behaviors of the MVDR and LCMV emerge: displacement of the main beam away from the source location and appearance of sidelobes. To explain these behaviors, recall that in the formulation of the optimization problems leading to the LCMV and MVDR, the array response towards the source direction is forced to the unity gain. We empirically verified that this constraint is satisfied (the maxima of both beampatterns correspond to values larger than one and the results presented in Fig. 2 are normalized with respect to the largest value). Physically, as the interference moves towards the target source, it becomes harder for the LCMV to satisfy two contradictory constraints: switching the array gain from zero to one. This fact results in instabilities that translate into the appearance of sidelobes and displacement of the maximum far from the interference. These sidelobes lead the beamformers to capture the white noise which spans the whole space. This physical interpretation is corroborated by our theoretical study above and the results provided in Fig. 1. Finally, it is obvious that when $\Delta\theta$ increases, the three filters perform relatively well especially in terms of noise removal. In terms of interference removal, the LCMV obviously outperforms both other beamformers. This suggests that it could be a very good candidate for interference removal when the the interference is placed far from the target source. However, one has to be very careful when using this filter because of the potential instabilities that it exhibits when this spatial separation is low. In the second setup ($T_{60} = 200$ ms), we show the resulting performance of the three filters in terms of output SNR and SIR for $\Delta\theta = 60^\circ$ and 10° for the frequency span 0 to 4 kHz. This effect is illustrated in Fig. 3 and is actually frequency dependent as we can see a wide dynamic range of both performance measures for the investigated frequency band. We also notice that the infinite gain in SIR achieved by the LCMV may come at the price of very low output SNR as compared to the other two filters, especially for the low frequency range (lower than 500 Hz). When we compare Figs. 3 (a) and (c) to Figs. 3 (b) and (d), respectively, we notice that when the interference is spatially close to the target source, a remarkable performance degradation is observed in terms of output SNR, especially for the LCMV filter, and in terms of output SIR, especially for the MVDR and matched filters.

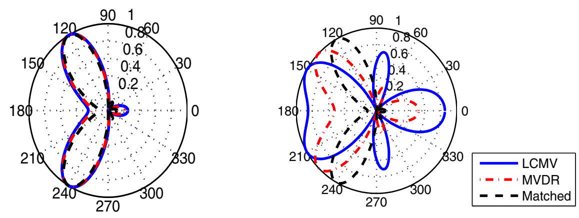
6. CONCLUSION

In this paper, we provided new insights into the MVDR and LCMV beamformers in the context of ambient noise and interference reduction. We demonstrated a new relationship between both filters in which the MVDR is shown to be a weighted linear combination of the LCMV and a matched filter (MVDR solution when only ambient noise overlaps with the target signal). Then, we analyzed the noise and interference reduction capabilities of these filters. Specifically, we developed new simplified expressions for their output SNR and SIR. These expressions theoretically demonstrate the tradeoff between noise and interference reduction. Indeed, total removal of the interference (by the LCMV) may result in the magnification of the ambient noise. Similarly, totally focussing on the ambient noise reduction (by the matched filter) may result in very poor output SIR. Our findings were finally corroborated by numerical evaluations in both anechoic and reverberant environments.



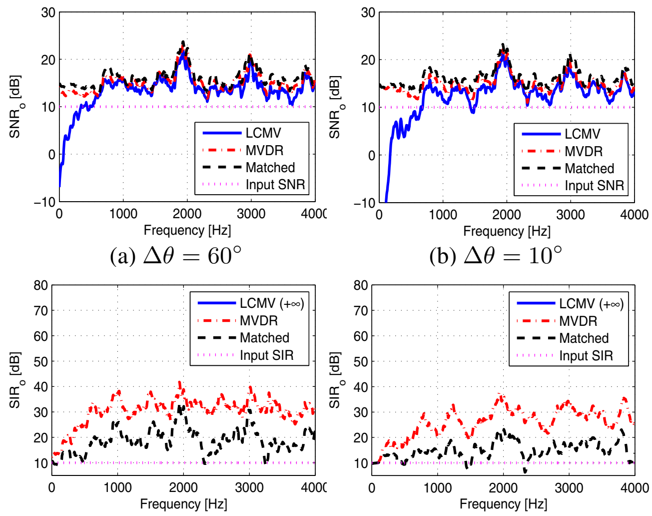
(a) Output SNR vs. $\Delta\theta$ (b) Output SIR vs. $\Delta\theta$

Figure 1: Effect of the angular separation between the interference and the target source on the performance of the MVDR, LCMV, and matched filters: anechoic room.



(a) $\Delta\theta = 60^\circ$ (b) $\Delta\theta = 10^\circ$

Figure 2: Beampatterns of the MDR, LCMV, and matched filters; the source is at 120° and the interference at $120^\circ - \Delta\theta$: anechoic room.



(a) $\Delta\theta = 60^\circ$ (b) $\Delta\theta = 10^\circ$

(c) $\Delta\theta = 60^\circ$ (d)$\Delta\theta = 10^\circ$

Figure 3: SNR and SIR at the output of the LCMV, MVDR, and matched filters: reverberant room.

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