A TWO-RAY SPECTRUM-APPROXIMATION APPROACH TO DOPPLER SPREAD ESTIMATION WITH ROBUSTNESS TO THE CARRIER FREQUENCY OFFSET

Mehrez Souden, Sofiène Affes, and Jacob Benesty

INRS-ÉMT, Université du Québec, Montréal, QC, Canada

ABSTRACT

We propose a new simple and accurate approach to estimate the Doppler spread which is a key parameter in the context of wireless communication systems. This new approach stems from the well known fact that the crosscorrelation of the channel is a weighted summation of monochromatic plane waves (or inverse Fourier transform of its power spectral density). In the case of Doppler spread, these plane waves are locally distributed around a main frequency which is nothing but the carrier frequency offset (CFO). This special feature accounts for the Taylor series expansions that we use herein to develop a two-ray spectrum approximate model. The resulting approximation allows us to determine a new simple and accurate closed-form estimator of the Doppler spread under the unique symmetry assumption on the channel's spectrum. Simulations illustrate the advantages of the proposed technique and its robustness to the CFO.

1. INTRODUCTION

The carrier frequency offset and Doppler spread are key parameters in the design of wireless communication systems [1]. While the CFO is caused by the real physical limitation of the oscillators at the receiver and the emitter to properly identify the carrier frequency and down-(or up-) convert the signals of interest, the Doppler spread is caused by the motion of one of the communicating ends with respect to the other.

A clear distinction has to be made herein between the *maximum Doppler frequency* and the *Doppler spread factor*. Indeed, while it is understood that the first is the maximum frequency shift caused by the Doppler phenomenon, the second stands for the standard deviation of the frequency around the CFO. Despite the direct relationship between both parameters [cf. (15) below], the deduction of one of them knowing the other requires the knowledge of the shape of the channel power spectral density (PSD). The proposed technique is able to determine the Doppler spread factor using the unique symmetry assumption on the channel's PSD.

Due to its importance in the design of adaptive communication systems, several techniques have been proposed so far for Doppler effect characterization. For instance, Hansen et al. proposed a maximum likelihood (ML) based approach in [2]. This approach assumes that the channel follows Jakes' model [1]. The maximization of the similarity between the PSD of the detected channel and a hypothetical one (corresponding to Jakes' model) leads to good estimates of the maximum Doppler frequency. Level crossing rate (LCR) based techniques have been also proposed in [3, 4]. These techniques take advantage of the direct relationship between the fading rate and the maximum Doppler frequency. Other approaches based on the channel covariance at different time lags have been also proposed, e.g., [5, 6, 7]. The latter techniques are known to be generally more efficient than LCR-based ones [5, 6]. To the best of our knowledge, most of these techniques were developed under the assumption of no CFO. This assumption is not practical since the CFO is inherent to the physical limitations of the oscillators at the receiver and the transmitter in a communication link. As an example, third generation partnership project (3GPP) standards tolerate a carrier frequency offset of up to 200 Hz at 2 GHz carrier frequency after RF down/up conversion (i.e., 0.1 ppm at the base station) [8]. In addition to the conventional parameters [e.g., signal-to-noise ratio (SNR), and observation window length], the performance of these wireless transceivers clearly depends on the CFO.

In this paper, we propose a new simple and accurate approach for the estimation of the Doppler spread factor which is robust to the CFO. This new approach stems from the well known fact that the crosscorrelation of the channel is a weighted summation of monochromatic plane waves (or inverse Fourier transform of its PSD). In the case of the Doppler spread, these plane waves are locally distributed around the CFO. We exploit these local frequency deviations jointly with a temporal aperture to develop a new two-ray approximate model that allows us to find a simple and accurate closed-form estimator of the Doppler spread factor without the knowledge of the channel'S PSD shape [1].

2. DATA MODEL AND ASSUMPTIONS

We consider the following data model:

[1] Robustness to the Doppler type and classification of the Doppler spread with respect to its spectrum shape in order to systematically retrieve the maximum Doppler frequency from the spread factor is currently under investigation.

$$x(t) = h(t) \cdot s(t) + v(t), \quad (1)$$

where $h(t)$ is a wide sense stationary process that models the channel experienced by the source signal $s(t)$ and $v(t)$ is an additive noise assumed to be Gaussian, zero-mean, and with temporally independent and identically distributed (i.i.d.) components. Here, we are interested in estimating the Doppler spread factor (and deducing the maximum Doppler frequency when the PSD shape is available) and the CFO. These parameters are denoted as σ_f ($f_{\rm d}$ stands for the maximum Doppler frequency) and $f_{\rm c}$, respectively. $s(t)$ is the training data assumed to be $s(t) = 1$ without loss of generality. We also assume that the normalized PSD of the channel, denoted $S(f)$ herein, is symmetrical around $f_{\rm c}$, which is commonly the case [1, 9].

The proposed approach is based on the observations covariance matrix, which is expressed under the above assumptions as:

$$\mathbf{R}_x = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{R}_h + \mathbf{R}_v, \quad (2)$$

where

$$\begin{aligned}
\mathbf{x}(t) &= [x(t_1+t) \ \cdots \ x(t_N+t)]^T, && (3)\\
\mathbf{R}_h &= E\{\mathbf{h}(t)\mathbf{h}^H(t)\}, && (4)\\
\mathbf{h}(t) &= [h(t_1+t) \ \cdots \ h(t_N+t)]^T, && (5)\\
\mathbf{R}_v &= E\{\mathbf{v}(t)\mathbf{v}^H(t)\} = \sigma_v^2 \mathbf{I}_N, && (6)\\
\mathbf{v}(t) &= [v(t_1+t) \ \cdots \ v(t_N+t)]^T, && (7)
\end{aligned}$$

where N is the number of time snapshots $(t_1, \cdots, t_N)$, σ_v^2 is the noise variance, $\mathbf{I}_N$ is the identity matrix of size $N \times N$, $(\cdot)^T$ is the transpose operator, and $(\cdot)^H$ is the trans-conjugate operator.

3. KEY PROPERTIES OF THE CHANNEL COVARIANCE MATRIX

In what follows, we will adopt the following notations: $\omega = 2\pi f$, $\omega_{\rm d} = 2\pi f_{\rm d}$, $\sigma_\omega = 2\pi\sigma_f$, $\omega_{\rm c} = 2\pi f_{\rm c}$ [2], $\tau_{lk} = t_l - t_k$ with $l,\ k \in \{1, \cdots, N\}$ (assuming without loss of generality that $l \geq k$). The crosscorrelation of the channel taken at two instants t_l and t_k is expressed using $S(\omega)$ as:

$$\begin{aligned}
r_h(t_l, t_k) &= r_h(\tau_{lk})\\
&= E\{h(t_l)h^*(t_k)\}\\
&= \frac{\sigma_h^2}{2\pi}\int_{-\pi}^{+\pi} S(\omega)e^{j\omega\tau_{lk}}d\omega\\
&= \frac{\sigma_h^2}{2\pi}\int_{\omega_{\rm c}-\omega_{\rm d}}^{\omega_{\rm c}+\omega_{\rm d}} S(\omega)e^{j\omega\tau_{lk}}d\omega \quad (8)
\end{aligned}$$

where σ_h^2 is the channel variance. It is clearly seen that the channel crosscorrelation (or covariance) at a given time lag τ_{lk} is the superposition of several monochromatic waves or rays which are distributed around the main ray at $\omega = \omega_{\rm c}$. In the case of Doppler spread, these frequency deviations are small. This fact accounts for the Taylor series expansions that we will use in the sequel. Before going further, we use a simple variable change in the integral above to obtain:

$$r_h(\tau_{lk}) = \sigma_h^2 e^{j\omega_{\rm c}\tau_{lk}} \int_{-\omega_{\rm d}}^{\omega_{\rm d}} \tilde{S}(\omega)e^{j\omega\tau_{lk}}d\omega, \quad (9)$$

where $\tilde{S}(\omega) = \frac{1}{2\pi}S(\omega+\omega_{\rm c})$ is in many practical cases symmetrical [1, 9]. Hence, the integral quantity in (9) is real and we can conclude from the equation above that the phase of $r_h(\tau_{lk})$ bears the information about $\omega_{\rm c}$, while its magnitude can be utilized to determine the maximum Doppler frequency $\omega_{\rm d}$.

Now, if we consider an observation window, i.e., N time snapshots of the channel that we write in a vector form as in (5) and calculate all the crosscorrelations, we obtain the following matrix:

$$\mathbf{R}_h = \sigma_h^2 \int_{-\omega_{\rm d}}^{\omega_{\rm d}} \tilde{S}(\omega)\mathbf{a}(\omega+\omega_{\rm c})\mathbf{a}^H(\omega+\omega_{\rm c})d\omega, \quad (10)$$

where

$$\mathbf{a}(\omega) = [e^{j\omega t_1} \ \cdots \ e^{j\omega t_N}]^T \quad (11)$$

and the (l, k)th entry of $\mathbf{R}_h$ is defined as:

$$[\mathbf{R}_h]_{lk} = r_h(\tau_{lk}). \quad (12)$$

The core idea of the proposed approach is actually based on the fact that the frequency deviations caused by the Doppler effect have generally small values. The problem of estimating $f_{\rm d}$ using (10) and (11) hence appears to be very similar to addressing the issue of angular spread estimation for locally scattered sources using an array of sensors [10, 11] where a spread source is approximated by two point sources spatially separated by twice the angular spread value. Similarly to [11], we expect the 2nd order Taylor series expansions of $\mathbf{a}(\omega)$ in (10) around $\omega_{\rm c}$ [4th-order Taylor series expansions of the matrix $\mathbf{a}(\omega)\mathbf{a}^H(\omega)$] to provide an accurate and simple estimate of σ_ω as half the spectral separation between two frequency rays approximating the channel's PSD. Indeed, we can express $\mathbf{R}_h$ as (cf. [11] for full details):

$$\mathbf{R}_h \approx \frac{\sigma_h^2}{2}\mathbf{A}(\omega_{\rm c}-\sigma_\omega, \omega_{\rm c}+\sigma_\omega)\mathbf{A}^H(\omega_{\rm c}-\sigma_\omega, \omega_{\rm c}+\sigma_\omega), \quad (13)$$

with

$$\mathbf{A}(\omega_{\rm c}-\sigma_\omega, \omega_{\rm c}+\sigma_\omega) = [\mathbf{a}(\omega_{\rm c}-\sigma_\omega) \ \ \mathbf{a}(\omega_{\rm c}+\sigma_\omega)] \quad (14)$$

In Fig. 1, we show the theoretical variations of the channel's PSD and the locations of both frequency rays at $\omega_{\rm c} \pm \sigma_\omega$ in the approximate form in (13) and (14) in both cases of Jakes

[2]Note that f and ω are related up to a 2π factor and so are σ_ω and σ_f. Hence, estimating one parameter is quivalent to estimating the other.

and three-dimensional (3-D) scattering models [1, 9]. Clearly, estimating the Doppler spread σ_ω amounts to localizing two rays separated by $2\sigma_\omega$ and symmetrically located around the CFO. Two important remarks can be drawn out:

Remark 1: The estimation of the Doppler spread factor, σ_ω, is robust to the CFO (i.e., built in). It is also robust and independent of the Doppler distribution. However, if we are interested in estimating f_d (e.g., to determine the velocity of the mobile terminal), we need a prior knowledge of the shape of the distribution. In such a case, we can use the straightforward relationship:

$$\sigma_\omega = \left(\int_{-\omega_\mathrm{d}}^{\omega_\mathrm{d}} \omega^2 \tilde{S}(\omega) d\omega\right)^{1/2}, \tag{15}$$

leading to:

$$\omega_\mathrm{d} = \begin{cases} \sqrt{2}\,\sigma_\omega & \text{for Jakes' model,} \\ \sqrt{3}\,\sigma_\omega & \text{for } 3-\mathrm{D} \text{ scattering model.} \end{cases} \tag{16}$$

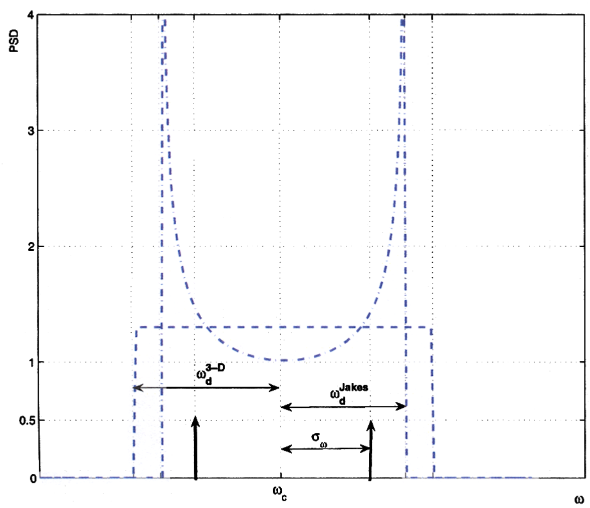

Fig. 1. Normalized theoretical channel PSD variations (dashed lines) and locations of the approximate two rays (at $\omega_c - \sigma_\omega$ and $\omega_c + \sigma_\omega$) in the particular cases of Jakes (U-shaped PSD) and 3-D (flat PSD) channel models.

Remark 2: As a built-in feature inherent to the proposed Doppler spread estimator, we are also able to estimate the CFO [3] f_c regardless of the frequency distribution (i.e., the PSD) provided that the latter is only symmetrical around a central ray (f_c).

In order to develop our Doppler spread estimator in the next section, we need to underline an important feature in the channel covariance matrix model given above. Indeed, in the particular case of no frequency spread (i.e., $\sigma_\omega = 0$), the matrix $\mathbf{R}_h$ reduces to:

$$\mathbf{R}_h = \mathbf{a}(\omega_\mathrm{c})\mathbf{a}^H(\omega_\mathrm{c}) \tag{17}$$

where $\mathbf{a}(\omega)$ is as defined in (11). We can see that in the absence of the Doppler spread, $\mathbf{R}_h$ is of rank one. Thus, one can use a point source localization algorithm to estimate the CFO (cf. [10] and references therein). In the presence of the Doppler spread, however, the case considered herein, we can conclude from the two-ray spectrum-approximation form in (13) and (14) that $\mathbf{R}_h$ is approximately of rank two (in essence due to the small channel frequency fluctuations around f_c resulting from the Doppler effect). We can hence use this feature to estimate the noise variance. Indeed, in practice, we only have an estimate of $\mathbf{R}_x$ (denoted $\hat{\mathbf{R}}_x$). Knowing that $\mathbf{R}_h$ is almost rank deficient and assuming that the noise is temporally i.i.d., we can estimate the noise variance, σ_v^2, by averaging over the last smallest eigenvalues of $\hat{\mathbf{R}}_x$, and hence exploit it in the Doppler spread estimator proposed below.

4. DOPPLER SPREAD ESTIMATION WITH CARRIER FREQUENCY OFFSET

The two-ray spectrum-approximation form in (13) and (14) can now be efficiently used to determine σ_f and f_c. We adapt the estimators that we proposed in our previous work [10] in the context of direction of arrival and angular spread estimation to the problem in hand. Indeed, according to the approximate form in (13) and (14), the (l,k)th entry of $\mathbf{R}_h$ can be expressed as:

$$r_h(t_l, t_k) \approx \sigma_h^2 \cos\left(2\pi\tau_{lk}\sigma_f\right) e^{j2\pi f_\mathrm{c}\tau_{lk}}. \tag{18}$$

Taking into account (2) and (18), the expression for the (l,k)th entry of $\mathbf{R}_x$ is:

$$r_x(t_l, t_k) \approx \sigma_h^2 \cos\left(2\pi\tau_{lk}\sigma_f\right) e^{j2\pi f_\mathrm{c}\tau_{lk}} + \delta(\tau_{lk}), \tag{19}$$

where $\delta(\tau_{lk})$ is the Dirac function. In practice, $\mathbf{R}_x$ is unavailable but can be estimated using T data samples as:

$$\hat{\mathbf{R}}_x = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}(t)\mathbf{x}^H(t) = \hat{\mathbf{R}}_h + \hat{\sigma}_v^2 \mathbf{I}_N. \tag{20}$$

The (l,k)th entry of this matrix, $\hat{r}_x(\tau_{lk})$, is a consistent estimate of $r_x(\tau_{lk})$. Note that when $t_l = t_k$, we obtain $r_x(0) = \sigma_h^2 + \sigma_v^2$ whose estimate is $\hat{r}_x(0) = \hat{\sigma}_h^2 + \hat{\sigma}_v^2$. As we stated previously, we average over the last smallest eigenvalues of $\hat{\mathbf{R}}_x$ to calculate $\hat{\sigma}_v^2$. Then, we deduce $\hat{\sigma}_h^2$ as:

$$\hat{\sigma}_h^2 \;=\; \hat{r}_x(0) - \hat{\sigma}_v^2. \tag{21}$$

[3] For lack of space, study of this inherent CFO estimation capability is beyond the scope of this paper.

To calculate the estimators of σ_f and f_c ($\hat{\sigma}_f$ and $\hat{f}_c$, respectively), we use $\hat{r}_x(\tau_{lk})$ for $l > k$ and minimize the following cost function:

$$J(f_c, \sigma_f) = \left|\hat{r}_x(\tau_{lk})/\hat{\sigma}_h^2 - \cos(2\pi\tau_{lk}\sigma_f)\, e^{j2\pi f_c \tau_{lk}}\right|^2, \tag{22}$$

with respect to f_c and σ_f. Straightforward calculations lead to the following estimators [4] [10]:

$$\hat{f}_c = \frac{1}{2\pi\tau_{lk}} \angle\{\hat{r}_x(\tau_{lk})\}, \tag{23}$$

$$\hat{\sigma}_f = \frac{\arccos\left(\Re\left\{\hat{r}_x(\tau_{lk})e^{-j2\tau_{lk}\pi\hat{f}_c}/\hat{\sigma}_h^2\right\}\right)}{2\pi\tau_{lk}}, \tag{24}$$

where $\angle\{\cdot\}$ is the angle operator and $\Re\{\cdot\}$ is the real part of the complex number between brackets. Based on the analogy with the angular spread estimation issue discussed earlier, we can easily infer from [10] that the smallest positive values of τ_{lk} lead to accurate estimates of f_c, while large values $\left[\text{near the first sign change of } \Re\left\{\hat{r}_x(\tau_{lk})e^{-j2\tau_{lk}\pi\hat{f}_c}\right\}\right]$ lead to accurate estimates of σ_f though it is clear that any of $\tau_{lk} > 0$ can be used in estimators (23) and (24).

Based on the knowledge of the channel's PSD shape and σ_f (or σ_ω), we can deduce f_d (or ω_d) using (15) [particularly (16) in the case of Jakes and 3-D scattering models].

For practical implementation, we only need to calculate the received signal covariances at $2p + 1 \leq N$ positive time lags (the $p+1$ smallest and p largest possible time lags, in our case, we empirically found that $p = 20$ provides accurate estimates). We use the first p time lags (> 0) to obtain p estimates of f_c; $\hat{f}_c$ being the average value of these estimates. Similarly we take the p largest time lags to estimate f_d. To estimate the noise variance, we form $\hat{\mathbf{R}}_x$ from the first $p + 1$ correlation estimates by using its Toeplitz structure. Then, we average over its smallest $L < (p - 1)$ (in our case $L = 10$) eigenvalues. This shows that the proposed procedure has a very low complexity. The overall complexity is dominated by the eigenvalue decomposition with a complexity of $O\left((p + 1)^3\right)$ operations.

5. SIMULATION RESULTS

For illustration purposes, we implement the data model in (1) using Jakes' model [1]. We run our simulations over $T = 1024$ data samples and average the obtained results over $MC = 10^3$ Monte-Carlo runs for all the investigated scenarios.

The performance index that we use is the normalized mean squared error (NMSE) between the estimated and actual parameter of interest (σ_ω). We compare the proposed approach to the so-called "hybrid method for Doppler spread estimation" (denoted as HAC herein) proposed in [7] which

[4] In the ideal case of no CFO, we force $\hat{f}_c = 0$ in (24).

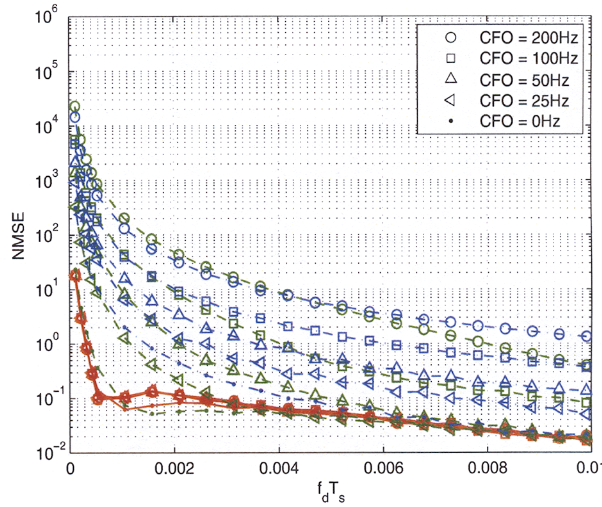

Fig. 2. NMSE(f_d) vs. f_dT_s, SNR = 0 dB, and f_c = 0, 25, 50, 100, and 200 Hz for the proposed estimator (solid/red), HAC (dashed/green), and ML (semi-dashed/blue).

is a combination of the methods proposed in [6] and [12], and also to the ML-based method proposed in [2].

In Fig. 2, we present the variations of the NMSE over the estimation of f_d with respect to f_dT_s in the cases $f_c = 0$, 25, 50, 100, and 200 Hz. f_dT_s is varied between 10^{-4} and 10^{-2} at an SNR = 0 dB.

It is clearly seen that the ML approach provides the poorest estimates since it is based on the similarity between the hypothetical spectrum at a CFO = 0 Hz and the actual one. The presence of a frequency shift due to the CFO deteriorates the performance of the ML. Similarly the performance of the HAC is affected but still better than ML for moderate CFO values. The proposed estimator is, in contrast, robust to the CFO and provides highly accurate results at very low Doppler spread values (even at around $f_dT_s \approx 10^{-3}$) typically encountered in vehicular high-data-rate communications. The HAC method is able to match the performance of our estimator only in the ideal case of no CFO.

Next, we choose $f_dT_s \approx 10^{-3}$ and $f_c = 0$, 25, 50, 100, and 200 Hz and assess in Fig. 3 the effect of the SNR variations on the proposed estimator, the ML and HAC approaches. The proposed estimator is able to achieve good estimates of σ_ω at very low SNR values (starting from 0 dB) while the ML and HAC clearly fail. Again the HAC method is able to match the performance of the proposed estimator only in the ideal case of no CFO.

It is worth noting that the estimation of the Doppler spread is more challenging at low f_dT_s values. In high data rate transmission systems (e.g., future-generation wireless networks), these values are more likely to be encountered even at

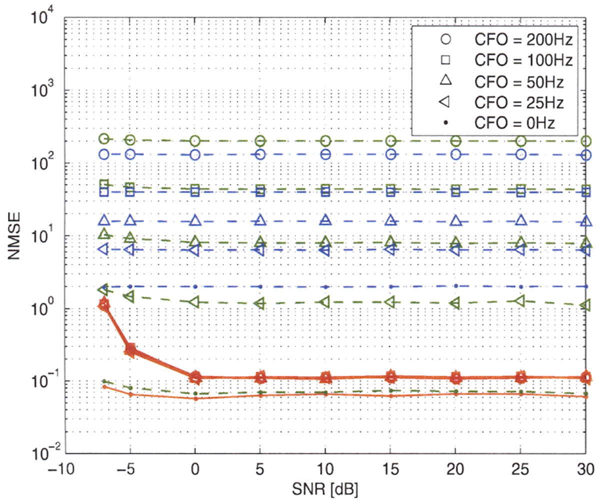

Fig. 3. NMSE($f_{\rm d}$) vs. SNR, $f_{\rm d}T_s \approx 10^{-3}$, and $f_{\rm c} = 0$, 25, 50, 100, and 200 Hz for the proposed estimator (solid/red), HAC (dashed/green), and ML (semi-dashed/blue).

a high range of Doppler spread or mobility since T_s could indeed have very low values. As shown above, with low computational complexity, the proposed approach provides accurate Doppler estimates at this very low range of $f_{\rm d}T_s$. This fact makes it a good candidate for communication systems that require robust Doppler estimates not only for mobile velocity estimation, but also for optimal adaptive processing in vehicular communications where the optimal adaptation step-size is directly related to the Doppler spread estimate [13].

6. CONCLUSION

In this paper, we proposed a new approach for Doppler spread estimation that exploits the typical behavior of relatively small channel frequency fluctuations around the main ray or CFO due to the Doppler effect. Taylor series expansions have led to a simple and accurate closed-form estimator of the Doppler spread at low complexity. This estimator is robust to the CFO as it inherently takes it into account. Simulation results have shown the great efficiency of the proposed approach even in adverse conditions: low SNR, small Doppler spread, high data rate transmissions, and large CFO.

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