

On the Performance Analysis of Composite Multipath/Shadowing Channels Using the \mathcal{G} -distribution

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Abstract—Composite multipath/shadowing fading environments are frequently encountered in different realistic scenarios. These channels are generally modeled as a mixture of Nakagami- m multipath fading and log-normal shadowing. The resulting composite probability density function (pdf) is unfortunately not in closed-form, thereby making the performance evaluation of communication links in these channels cumbersome. In this paper, we propose to model composite channels by the very general \mathcal{G} -distribution. This pdf arises when the log-normal shadowing is substituted by the Inverse-Gaussian one. This substitution will prove to be very accurate for several shadowing conditions. In this paper we conduct an exhaustive performance evaluation of communication systems operating in these channels. Our study starts by deriving a closed-form expression for the outage probability. Then, we derive the moment generating function of the \mathcal{G} -distribution, hence facilitating the calculation of average bit error probabilities. We also derive closed-form expressions for the channel capacity for three adaptive transmission techniques, namely, i) optimal rate adaptation with constant power, ii) optimal power and rate adaptation, and iii) channel inversion with fixed rate. The different expressions that will be provided are of great importance in assessing the performance of communication systems in composite channels.

Index Terms—Outage probability, information rates, fading channels, adaptive transmission techniques, log normal, Nakagami distribution and composite distributions.

I. INTRODUCTION

Mixtures of multipath fading and shadowing are frequently encountered in several scenarios. This is particularly the case for communication systems with low mobility or stationary users. In such configurations, the receiver can not mitigate the multipath fading effect by averaging and is subject to the instantaneous composite multipath/shadowed signal. A composite distribution arises therefore as the perfect statistical characterization of the signal to noise ratio in these channels. Several composite models were presented in the literature (see [1] and the references therein), like for instance, the shadowed Nakagami fading channel [2], which is a generalization of the Rayleigh-lognormal model (called also the Suzuki model) [3]-[4], and consists of a mixture of Nakagami- m multipath fading and log-normal shadowing.

The main drawback of the shadowed Nakagami fading model is that the composite probability density function (pdf) is not

in closed-form thereby making the performance evaluation (such as average error probabilities, outage probabilities and channel capacity) of communication links in these channels cumbersome. Attempts have been made to obtain a practical closed-form composite distribution. We can cite, for instance, the well known \mathcal{K} -distribution [5] and its generalized version [6]. The \mathcal{K} -distribution is obtained by substituting the gamma shadowing to the log-normal one. This distribution proved to be particularly useful in evaluating the performance of composite channels [7]-[10]. Recently, the Inverse-Gaussian pdf was proposed as a substitute to the log-normal one [11]. The authors proved that a composite Rayleigh-Inverse Gaussian distribution approximates the Suzuki distribution more accurately than the \mathcal{K} -distribution.

In this paper, we consider the more general Nakagami-Inverse Gaussian model. We demonstrate that this combination gives birth to a closed-form composite distribution called the \mathcal{G} -distribution. This distribution was first proposed in [12] in the context of Synthetic Aperture Radar (SAR) image modeling. In this paper, we derive several important tools for the performance evaluation of communications links in such channels. Our study starts by deriving a closed-form expression for the outage probability. Then, we derive the moment generating function (MGF) of the \mathcal{G} -distribution, hence making the average bit error probabilities in this type of channels (with and without diversity combining) easy to compute. We also derive closed-form expressions for the channel capacity with different adaptive transmission techniques.

The remainder of the paper is organized as follows. In section II, we present the \mathcal{G} -distribution and some of its properties. In section III, we give the expression for the outage probability. Section IV deals with derivation of the MGF and average bit error probabilities. In section V, we give a closed-form expression for the capacity of three adaptive transmission techniques namely, i) optimal rate adaptation with constant power, ii) optimal power and rate adaptation, and iii) channel inversion with fixed rate. Section VI provides some selected numerical results to illustrate the derived formulas and validates the newly developed analytical expressions via computer simulations. Section VII concludes the paper with a summary

of the main results.

II. THE \mathcal{G} -DISTRIBUTION

A. The Probability Density Function of the Composite Envelope

In a composite Nakagami-lognormal channel, the probability density function of the envelope X is

$$f_X(x) = \int_0^{+\infty} f_{X/Y}(x/Y=y)f_Y(y)dy, \quad (1)$$

where $f_{X/Y}$ is the Nakagami- m multipath fading distribution and is given by

$$f_{X/Y}(x/Y=y) = \frac{2m^m x^{2m-1} \exp(-\frac{mx^2}{y})}{\Gamma(m)y^m}, \quad x > 0, \quad (2)$$

where $\Gamma(\cdot)$ is the gamma function [20] and m is generally an arbitrary number superior to 0.5. However, in our performance study, this parameter will be restricted to integer values for analytical tractability.

In (1), $f_Y(y)$ is the log-normal shadowing distribution, i.e.,

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma y} \exp\left(-\frac{(\ln(y) - \mu)^2}{2\sigma^2}\right), \quad y > 0, \quad (3)$$

where μ and σ are, respectively, the mean and the standard deviation of $\ln(y)$. The resulting composite distribution $f_X(\cdot)$ is unfortunately not in closed-form, hence making the performance evaluation of communications links over such channels very challenging. In order to obtain a more tractable composite distribution, and as it was done in [11], the log-normal shadowing is approximated by the Inverse-Gaussian (IG) distribution which is given by

$$f_Y(y) = \sqrt{\frac{\lambda}{2\pi}} y^{-\frac{3}{2}} \exp\left(-\frac{\lambda(y - \theta)^2}{2\theta^2 y}\right), \quad y > 0, \quad (4)$$

where λ and θ can be linked to μ and σ by the moment matching technique as follows¹

$$\begin{cases} \lambda = \frac{\exp(\mu)}{2 \sinh(\frac{\sigma^2}{2})}, \\ \theta = \exp\left(\mu + \frac{\sigma^2}{2}\right). \end{cases}$$

Substituting (2) and (4) in (1) and using [20, Eq. (3.471.9)], we find that this substitution results in a closed-form composite distribution given by

$$f_X(x) = \left(\frac{\lambda}{\theta^2}\right)^{m+\frac{1}{2}} \sqrt{\frac{\lambda}{2\pi}} \frac{4m^m \exp(\frac{\lambda}{\theta}) x^{2m-1}}{\Gamma(m) (\sqrt{g(x)})^{m+\frac{1}{2}}} K_{m+\frac{1}{2}}(\sqrt{g(x)}), \quad (5)$$

where $g(x) = \frac{2\lambda}{\theta^2}(mx^2 + \frac{\lambda}{2})$ and $K_\nu(\cdot)$ is the modified Bessel function of the second kind of order ν [20]. This pdf was first discovered in [12] where it was called the \mathcal{G} -distribution (in

[12], it is referred to as \mathcal{G}_A). If $m = 1$, this distribution reduces to the Rayleigh-Inverse Gaussian model that was considered in [11]. Previously, the gamma pdf was used as a substitute to the log-normal one. The resulting composite pdf is the generalized \mathcal{K} -distribution. In [11], the authors demonstrated that a composite Rayleigh-Inverse Gaussian distribution can better describe a composite Rayleigh-lognormal channel. This fact is further confirmed later in our numerical examples.

B. The Probability Density Function of the Instantaneous Composite SNR

Using [13, Eq. (2.3)], the pdf of the composite instantaneous signal-to-noise power ratio (SNR) $f_\gamma(\gamma)$, can be easily deduced from (5) as

$$f_\gamma(\gamma) = A \frac{\gamma^{m-1}}{(\sqrt{\alpha + \beta\gamma})^{m+\frac{1}{2}}} K_{m+\frac{1}{2}}(b\sqrt{\alpha + \beta\gamma}), \quad (6)$$

where the following constants have been used:

$$\begin{cases} A = \frac{(\lambda\bar{\gamma})^{\frac{1+2m}{4}}}{\Gamma(m)} \sqrt{\frac{2\lambda}{\pi\theta}} \exp(\frac{\lambda}{\theta}) \left(\frac{m}{\bar{\gamma}}\right)^m, \quad b = \frac{1}{\theta} \sqrt{\frac{\lambda}{\bar{\gamma}}}, \\ \alpha = \lambda\bar{\gamma}, \quad \beta = 2m\theta. \end{cases}$$

Note that if a maximum ratio combiner with M i.i.d. branches is used at the receiver, then the distribution of the instantaneous SNR at the output of this combiner can be readily obtained from (6) by substituting m with Mm and $\bar{\gamma}$ with $M\bar{\gamma}$ (refer to [14] for a similar analysis treating the \mathcal{K} -distributed fading). Consequently, all the following performance study applies also if maximum ratio combining (in i.i.d. fading) is employed at the receiver.

C. Moments and Amount of Fading

Using [20, Eq. (6.596.3)], the k th moment of the output SNR can be found to be given by

$$\begin{aligned} E[\gamma^k] &= A \int_0^{+\infty} \frac{\gamma^{m+k-1}}{(\sqrt{\alpha + \beta\gamma})^{m+\frac{1}{2}}} K_{m+\frac{1}{2}}(b\sqrt{\alpha + \beta\gamma}) d\gamma \\ &= \sqrt{\frac{2\lambda}{\pi\theta}} e^{\frac{\lambda}{\theta}} \left(\frac{\bar{\gamma}}{m}\right)^k \frac{\Gamma(m+k)}{\Gamma(m)} K_{k-\frac{1}{2}}\left(\frac{\lambda}{\theta}\right), \end{aligned} \quad (7)$$

which yields the following Amount of Fading (AF)

$$AF = \frac{E[\gamma^2]}{E[\gamma]^2} - 1 = \left(\frac{1}{m} + 1\right) \left(\frac{\theta}{\lambda} + 1\right) - 1. \quad (8)$$

The AF ranges from $\frac{\theta}{\lambda}$ (for $m = +\infty$) to $3\frac{\theta}{\lambda} + 2$ (for $m = \frac{1}{2}$).

III. OUTAGE PROBABILITY

The outage probability is an important performance measure of communication links operating over fading channels. It is defined as the probability that the output SNR falls below a given threshold γ_{th} , i.e.,

$$P_{out} = \int_0^{\gamma_{th}} f_\gamma(\gamma) d\gamma = F_\gamma(\gamma_{th}) - F_\gamma(0), \quad (9)$$

where $F_\gamma(\cdot)$ is the primitive of the instantaneous SNR's pdf and is defined as

$$F_\gamma(\gamma) = \int f_\gamma(\gamma) d\gamma = AI_{m,m}(\gamma, \beta, \alpha), \quad (10)$$

¹Note that, in [11], the authors use different matching equations. Indeed, they match the moments of the Suzuki distribution with the moments of the Rayleigh-Inverse Gaussian one. In our case, we choose to match the moments of the IG and the log-normal distribution and this has the advantage of leading to simpler matching equations between (λ, θ) and (μ, σ) .

where

$$I_{p,q}(x, y, z) = \int x^{q-1} \frac{K_{p+\frac{1}{2}}(b\sqrt{xy+z})}{(\sqrt{xy+z})^{p+\frac{1}{2}}} dx. \quad (11)$$

Using the results of Appendix I, $F_\gamma(\gamma)$ can be written in closed-form as

$$F_\gamma(\gamma) = -A\Gamma(m) \sum_{k=1}^m \frac{2^k \gamma^{m-k}}{(\beta b)^k (m-k)!} \frac{K_{m-k+\frac{1}{2}}(b\sqrt{\alpha+\beta\gamma})}{(\sqrt{\alpha+\beta\gamma})^{m+\frac{1}{2}-k}}. \quad (12)$$

Consequently, the outage probability is given by

$$P_{\text{out}} = 1 - A\Gamma(m) \sum_{k=1}^m \frac{2^k \gamma_{\text{th}}^{m-k}}{(\beta b)^k (m-k)!} \frac{K_{m-k+\frac{1}{2}}(b\sqrt{\alpha+\beta\gamma_{\text{th}}})}{(\sqrt{\alpha+\beta\gamma_{\text{th}}})^{m-k+\frac{1}{2}}}. \quad (13)$$

IV. AVERAGE BIT ERROR PROBABILITY

The average bit error probability (BEP) constitutes probably the most important performance measure of a digital communication system. Unfortunately, the average BEP is generally not easy to find in closed-form. However, it was shown in [13], that the MGF can be used to obtain the average BEP of any kind of modulation (with and without diversity) either in closed-form or in the form of a single finite-range integral. For instance, for differentially coherent detection of phase-shift-keying (DPSK) or noncoherent detection of orthogonal frequency-shift-keying (FSK), the average BEP can be written as [13]

$$P_b(E) = C_1 M(a_1), \quad (14)$$

where $M(\cdot)$ is the MGF and C_1 and a_1 are constants that depend on the modulation.

The MGF is therefore a key tool that needs to be derived. In Appendix II, we prove that the MGF corresponding to the \mathcal{G} -distribution can be written in closed-form as

$$M(s) = 1 + m \sum_{k=0}^{m-1} \frac{(-1)^{k+1} C_{m-1}^k}{(k+1)!} \sum_{p=0}^k C_k^p \left(2\sqrt{\frac{\alpha s}{\beta}} \right)^{k+1-p} \\ \times \Gamma[k+p+1] H_{-(k+p+1)} \left(\frac{b}{2} \sqrt{\frac{\beta}{s}} + \sqrt{\frac{s\alpha}{\beta}} \right), \quad (15)$$

where C_k^p is the binomial coefficient and $H_\nu(x)$ is the Hermite function of order ν [21].

V. CHANNEL CAPACITY

Spectral efficiency of adaptive transmission techniques has attracted a rising concern in the last decade. This interest stems from the fact that Shannon's channel capacity represents the upper bound for the data rate achievable in a transmission with an arbitrary small error probability, and, as such, serves as an ultimate performance measure of communication systems. The evaluation of the capacity of fading channels mainly started with Lee's paper [15], in which he analyzed the capacity of Rayleigh fading channels under the optimal rate constant power policy. Since then, several results on wireless channel capacity became available. In [16], Alouini

and Goldsmith extended the work of Lee by examining the capacity of Rayleigh fading channels under different adaptive transmission techniques and different configurations. Other fading channels like Rician, Hoyt, Nakagami, Weibull and \mathcal{K} fading channels were studied in [17], [18], [19] and [10]. Here, we present closed-form expressions for the capacity with different adaptive transmission techniques for the \mathcal{G} -distributed fading channels.

A. Optimal Rate Adaptation with Constant Transmit Power

Under the optimal rate constant power (ora) policy the capacity is known to be given by [16]

$$\langle C \rangle_{\text{ora}} = \int_0^{+\infty} \ln(1+\gamma) f_\gamma(\gamma) d\gamma = - \int_0^{+\infty} \frac{F_\gamma(\gamma)}{1+\gamma} d\gamma. \quad (16)$$

Substituting $F_\gamma(\gamma)$ with the expression obtained above, then using the change of variable $v = 1 + \gamma$ and applying the binomial expansion, we obtain after some manipulations

$$\langle C \rangle_{\text{ora}} = A\Gamma(m) \sum_{k=1}^m \frac{2^k (-1)^{m-k}}{(\beta b)^k (m-k)!} (R_{m-k}(\beta, \alpha - \beta) \\ - \sum_{j=1}^{m-k} C_{m-k}^j (-1)^j I_{m-k,j}(1, \beta, \alpha - \beta)), \quad (17)$$

where $R_n(\cdot, \cdot)$ is given by

$$R_n(x, y) = \int_1^{+\infty} \frac{K_{n+\frac{1}{2}}(b\sqrt{xv+y})}{v(\sqrt{xv+y})^{n+\frac{1}{2}}} dv, \quad (18)$$

and is obtained in closed-form in Appendix III.

B. Optimal Simultaneous Power and Rate Adaptation

For optimal power and rate adaptation (opra), the capacity is known to be given by [16]

$$\langle C \rangle_{\text{opra}} = \int_{\gamma_0}^{+\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) f_\gamma(\gamma) d\gamma = - \int_{\gamma_0}^{+\infty} \frac{F_\gamma(\gamma)}{\gamma} d\gamma. \quad (19)$$

By replacing $F_\gamma(\gamma)$ with its expression, we obtain

$$\langle C \rangle_{\text{opra}} = A\Gamma(m) \left(\frac{2^m}{(\beta b)^m} R_0(\gamma_0 \beta, \alpha) \right. \\ \left. - \sum_{k=1}^{m-1} \frac{2^k I_{m-k, m-k}(\gamma_0, \beta, \alpha)}{(\beta b)^k (m-k)!} \right). \quad (20)$$

The variable γ_0 is the optimal cutoff and is the solution of the following equation

$$\int_{\gamma_0}^{+\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma} \right) f_\gamma(\gamma) d\gamma = 1. \quad (21)$$

Note that by using the same techniques as previously, this integral can be expressed in closed-form. However γ_0 can not be solved in closed-form and the last equation has to be evaluated numerically.

C. Channel Inversion with Fixed Rate

1) *Total Channel Inversion*: The capacity for this scheme is known to be given by [16]

$$\langle C \rangle_{\text{cif}} = \ln \left(1 + \frac{1}{\int_0^{+\infty} \frac{1}{\bar{\gamma}} f_{\gamma}(\gamma) d\gamma} \right). \quad (22)$$

For $m \geq 2$, the integral in the logarithm can be expressed in terms of $I_{m,m-1}(0, \beta, \alpha)$, yielding the following expression

$$\langle C \rangle_{\text{cif}} = \ln \left(1 + \bar{\gamma} \frac{(m-1)}{m} \frac{\lambda}{\lambda + \theta} \right). \quad (23)$$

Note that, for Rayleigh fading, i.e. $m = 1$, the capacity with total channel inversion tends to zero. This is because the integral inside the logarithm will diverge.

2) *Truncated Channel Inversion*: The capacity of this scheme is given by [16]

$$\langle C \rangle_{\text{tcif}} = \ln \left(1 + \frac{1}{\int_{\gamma_0}^{+\infty} \frac{1}{\bar{\gamma}} f_{\gamma}(\gamma) d\gamma} \right) (1 - P_{\text{out}}), \quad (24)$$

which is readily obtained using the previously derived results and the results presented in the appendix.

VI. NUMERICAL RESULTS

Fig. 1 shows the probability density functions of the log-normal, the Inverse-Gaussian, and the Gamma distributions in a frequent heavy shadowing environment (refer to [1] for more information on this model). This figure shows clearly that the IG-distribution can be used as a precise substitute for the log-normal one.

In the next examples, we compare the closed-form expressions that we have developed (referred to in the figures as analytical formulae) with numerical integration (referred to as simulation). This comparison is conducted for two shadowing scenarios, namely the average shadowing environment and the frequent heavy shadowing one. Throughout our simulations the parameter m is arbitrarily set to 5.

Fig. 2 depicts the outage probability versus the average SNR $\bar{\gamma}$ for $\gamma_0 = 5$ dB. As expected, the outage probability increases as the shadowing becomes more pronounced. The average bit error probability for DPSK ($C_1 = \frac{1}{2}$ and $a_1 = 1$ in (14)) and noncoherent frequency shift keying ($C_1 = \frac{1}{2}$ and $a_1 = \frac{1}{2}$ in (14)) is also illustrated in Fig. 3. Here also the performance is degraded in the frequent heavy shadowing environment.

Fig. 4 shows also that compared to optimal power and rate adaptation, transmission with optimal rate adaptation suffers capacity penalty at low SNR only. However, as $\bar{\gamma}$ increases, the two policies will provide the same capacity. The same holds for truncated channel inversion and total channel inversion, however due to the space limitations, these results are not shown here.

VII. CONCLUSION

Composite multipath/shadowing fading environments are frequently encountered in several realistic scenarios. In this paper, hinging on the fact that the Inverse-Gaussian distribution

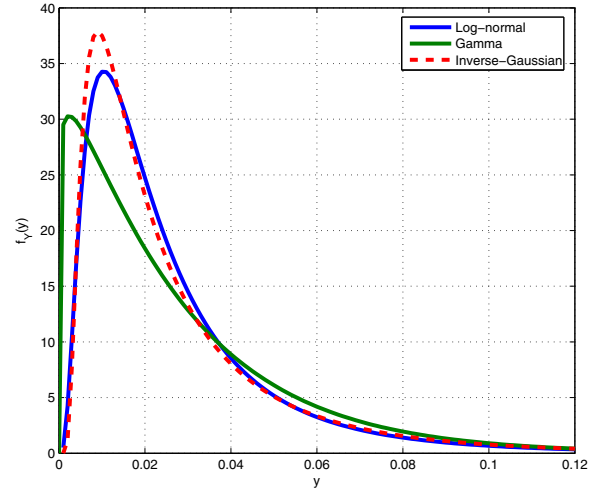


Fig. 1. Pdf of the log-normal, the inverse-Gaussian, and the gamma distribution for frequent heavy shadowing ($\mu = -3.914$ and $\sigma = 0.806$).

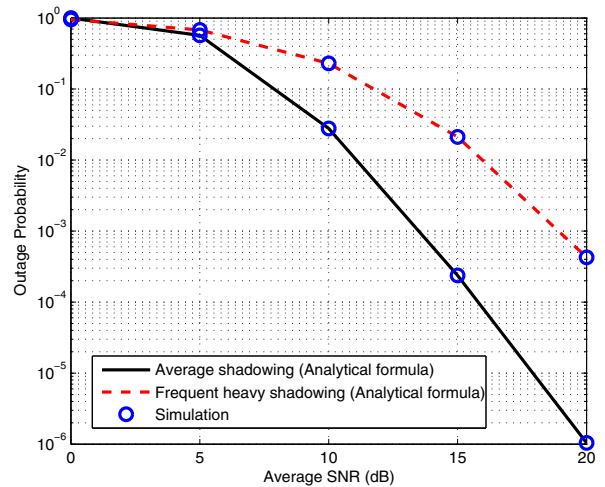


Fig. 2. Outage probability for $\gamma_0 = 5$ dB in average ($\mu = -0.115$ and $\sigma = 0.161$) and frequent heavy shadowing environments ($\mu = -3.914$ and $\sigma = 0.806$).

accurately approximates the log-normal one, we have proposed to use the Nakagami-Inverse Gaussian composite model as a substitute for the log-normally shadowed Nakagami fading. The resulting distribution, the \mathcal{G} -distribution, has the advantage of being in closed-form, thereby facilitating the performance evaluation of communication links over composite channels. In this study, several key results have been presented, including the outage probability, average error probabilities, and the capacity for different adaptive transmission techniques. The expressions that we have provided are of great importance in assessing the performance of communication systems over composite channels.

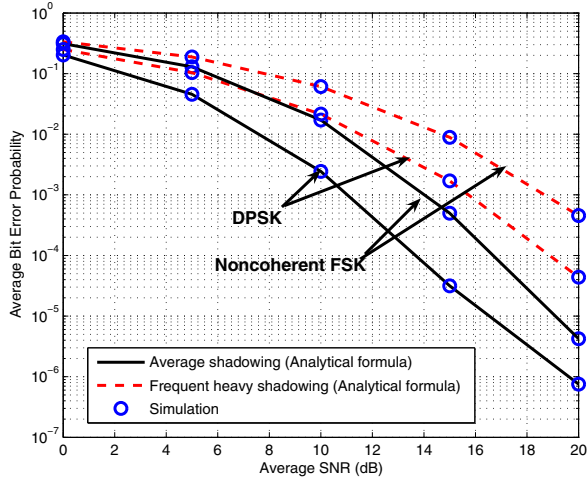


Fig. 3. Average bit error probability for DPSK and noncoherent FSK in average ($\mu = -0.115$ and $\sigma = 0.161$) and frequent heavy shadowing environments ($\mu = -3.914$ and $\sigma = 0.806$).

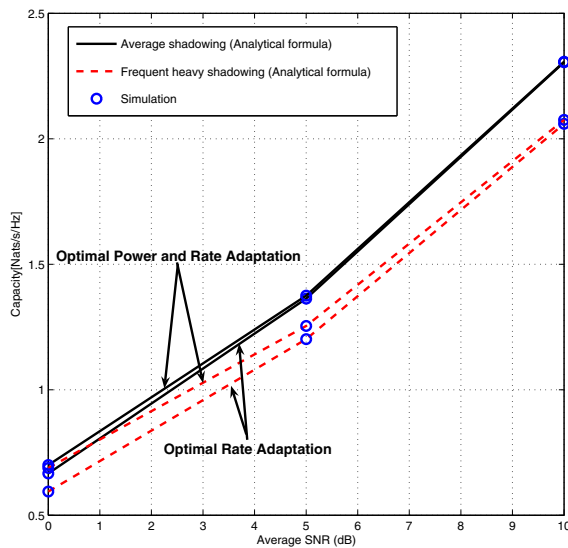


Fig. 4. Capacity for ORA and OPRA policies versus SNR in average ($\mu = -0.115$ and $\sigma = 0.161$) and frequent heavy shadowing environments ($\mu = -3.914$ and $\sigma = 0.806$).

APPENDIX A EVALUATION OF $I_{p,q}(x, y, z)$

Define $I_{p,q}(x, y, z)$ as

$$I_{p,q}(x, y, z) = \int x^{q-1} \frac{K_{p+\frac{1}{2}}(b\sqrt{xy+z})}{(\sqrt{xy+z})^{p+\frac{1}{2}}} dx. \quad (25)$$

Using the fact that

$$\frac{d(x^{-\mu} K_{\mu}(x))}{dx} = -x^{-\mu} K_{\mu+1}(x), \quad (26)$$

and integrating by part we obtain

$$I_{p,q}(x, y, z) = -\frac{2x^{q-1} K_{p-\frac{1}{2}}(b\sqrt{xy+z})}{yb (\sqrt{xy+z})^{p-\frac{1}{2}}} + \frac{2(q-1)}{yb} I_{q-1,p-1}(x, y, z). \quad (27)$$

Iterating over this equation, we finally find that

$$I_{p,q}(x, y, z) = -(q-1)! \sum_{k=1}^q \frac{2^k x^{q-k}}{(yb)^k (q-k)!} \frac{K_{p-k+\frac{1}{2}}(b\sqrt{xy+z})}{(\sqrt{xy+z})^{p+\frac{1}{2}-k}}. \quad (28)$$

APPENDIX B

DERIVATION OF THE MOMENT GENERATING FUNCTION

The MGF is given by

$$M(s) = A \int_0^{+\infty} \exp(-s\gamma) \gamma^{m-1} \frac{K_{m+\frac{1}{2}}(b\sqrt{\alpha+\beta\gamma})}{(\sqrt{\alpha+\beta\gamma})^{m+\frac{1}{2}}} d\gamma = (-1)^{m-1} A \frac{d^{m-1} G_m(s)}{ds^{m-1}}, \quad (29)$$

where $G_m(s)$ is defined as follows

$$G_m(s) = \int_0^{+\infty} \exp(-s\gamma) \frac{K_{m+\frac{1}{2}}(b\sqrt{\alpha+\beta\gamma})}{(\sqrt{\alpha+\beta\gamma})^{m+\frac{1}{2}}} d\gamma. \quad (30)$$

Applying an integration by part, we find that $G_m(s)$ satisfies the following recursion formula

$$G_m(s) = \frac{2}{b\beta} \frac{K_{m-\frac{1}{2}}(b\sqrt{\alpha})}{(\sqrt{\alpha})^{m-\frac{1}{2}}} - \frac{2s}{b\beta} G_{m-1}(s). \quad (31)$$

Iterating over this equation, we obtain

$$G_m(s) = \sum_{k=1}^m (-1)^{k-1} \frac{2^k s^{k-1}}{(b\beta)^k} \frac{K_{m-k+\frac{1}{2}}(b\sqrt{\alpha})}{(\sqrt{\alpha})^{m-k+\frac{1}{2}}} + (-1)^m \frac{2^m s^m}{(b\beta)^m} G_0(s), \quad (32)$$

where $G_0(s)$ is given by

$$G_0(s) = \sqrt{\frac{\pi}{2b}} \int_0^{+\infty} \exp(-s\gamma) \frac{e^{-b\sqrt{\beta\gamma+\alpha}}}{\sqrt{\beta\gamma+\alpha}} d\gamma. \quad (33)$$

By plugging the expression of $G_m(s)$ in the MGF, the latter will be given by

$$M(s) = 1 - \frac{\Gamma(m+1)A2^m}{(b\beta)^m} \sum_{k=0}^{m-1} C_{m-1}^k \frac{s^{k+1}}{(k+1)!} G_0^{(k)}(s). \quad (34)$$

The k th derivative of G_0 can be calculated as follows

$$G_0^{(k)}(s) = (-1)^k \sqrt{\frac{\pi}{2b}} \int_0^{+\infty} \gamma^k \exp(-s\gamma) \frac{e^{-b\sqrt{\beta\gamma+\alpha}}}{\sqrt{\beta\gamma+\alpha}} d\gamma, \quad (35)$$

which, after some manipulations, leads to

$$G_0^{(k)}(s) = \frac{(-1)^k}{\beta^{k+1}} \sqrt{\frac{2\pi}{b}} e^{-b\sqrt{\alpha}} \sum_{p=0}^k C_k^p (2\sqrt{\alpha})^{k-p} \times \int_0^{+\infty} y^{k+p} e^{-\frac{sy^2}{\beta}} e^{-y(b+\frac{2s\sqrt{\alpha}}{\beta})} dy. \quad (36)$$

Using [20, Eq. (3.462.1)] with [21], the last equation can be written as

$$G_0^{(k)}(s) = \frac{(-1)^k}{\beta^{k+1}} \sqrt{\frac{2\pi}{b}} e^{-b\sqrt{\alpha}} \sum_{p=0}^k C_k^p (2\sqrt{\alpha})^{k-p} \left(\frac{s}{\beta}\right)^{-\frac{k+p+1}{2}} \times \Gamma[k+p+1] H_{-(k+p+1)} \left(\frac{b\sqrt{\beta}}{s} + \sqrt{\frac{s\alpha}{\beta}} \right), \quad (37)$$

where $H_\nu(x)$ is the Hermite function.

APPENDIX C EVALUATION OF $R_n(x, y)$

Define $R_n(x, y)$ as

$$R_n(x, y) = \int_1^{+\infty} \frac{K_{n+\frac{1}{2}}(b\sqrt{xy+y})}{v(\sqrt{xy+y})^{n+\frac{1}{2}}} dv. \quad (38)$$

Using the change of variable $X = \sqrt{xy+y}$, we obtain

$$R_n(x, y) = 2 \int_{\sqrt{x+y}}^{+\infty} \frac{K_{n+\frac{1}{2}}(bX)}{\sqrt{X^2-y} X^{n-\frac{1}{2}}} dX. \quad (39)$$

Using the fact that $K_{n+\frac{1}{2}}(bX)$ can be written as

$$K_{n+\frac{1}{2}}(bX) = \sqrt{\frac{\pi}{2bX}} \exp(-bX) \sum_{l=0}^n \frac{\Gamma(n+1+l)(2bX)^{-l}}{\Gamma(n+1-l)\Gamma(l+1)}, \quad (40)$$

we obtain

$$R_n(x, y) = \sqrt{\frac{2\pi}{b}} \sum_{l=0}^n \frac{\Gamma(n+1+l)(2b)^{-l}}{\Gamma(n+1-l)\Gamma(l+1)} H_{n+l}(\sqrt{y}, \sqrt{x+y}), \quad (41)$$

where

$$H_l(a, c) = \int_c^{+\infty} \frac{\exp(-bX)}{(X^2 - a^2)X^l} dX. \quad (42)$$

This integral can be solved by applying partial fraction decomposition

$$\frac{a^{l+1}}{(X^2 - a^2)X^l} = \frac{1}{2} \left[\frac{1}{X-a} - \frac{(-1)^l}{X+a} \right] - \sum_{j=1}^{\frac{l+t}{2}} \frac{1}{X^{2j-t} a^{-2j+t+1}}, \quad (43)$$

where $t = \frac{1-(-1)^l}{2}$. Two cases must be distinguished here. The first one corresponds to the event $a \neq 0$, in this case, $H_l(a, c)$ will be given by

$$H_l(a, c) = \frac{e^{-ab}\Gamma[0, b(c-a)] - (-1)^l e^{ab}\Gamma[0, b(c+a)]}{2a^{l+1}} - \sum_{j=1}^{\frac{l+t}{2}} \frac{E_{2j-t}(bc)}{c^{2j-t-1} a^{l-2j+t+2}}, \quad (44)$$

where $\Gamma(\cdot, \cdot)$ is the incomplete Gamma function and $E_n(z) = \int_1^{+\infty} \frac{e^{-zt}}{t^n} dt$ is the n th order exponential integral function [20]. The second case corresponds to $a = 0$ and leads to $H_l(0, c) = \frac{E_{l+2}(bc)}{c^{l+1}}$.

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