On the Capacity of Generalized-$\mathcal{K}$ Fading Channels

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***Abstract*— This paper investigates the capacity of generalized-$\mathcal{K}$ fading channels. This very general model describes accurately composite multipath/shaodwing fading channels which are widely encountered in real-world environments. We derive closed-form expressions for three adaptive transmission techniques, namely, i) optimal rate adaptation with constant power, ii) optimal power and rate adaptation, and iii) channel inversion with fixed rate. The analytical expressions obtained match perfectly the results obtained by computer simulations. These expressions provide a good tool to assess the spectral efficiency of the aforementioned adaptive transmission techniques over composite channels.**

***Index Terms*— Shannon capacity, adaptive transmission techniques and Generalized-$\mathcal{K}$ distribution.**

I. Introduction

Wireless channels are commonly modeled as a mixture of multipath fading and shadowing. In such settings, the receiver is subject to the composite multipath/shadowed signal. This is generally the case for communication systems with low mobility or stationary users [1]-[2]. This model is also encountered in certain land-mobile satellite systems (see [3] and the references therein). Several composite models were proposed in the literature. Two of these are widely used and are the Suzuki channel [2] and its generalization, the Nakagami-m shadowed channel [1]. The main drawback of these two models is that the composite probability density function (pdf) is not in closed-form, thereby making the performance evaluation of communication links over these channels cumbersome. In order to obtain a practical closed-form composite distribution, the log-normal shadowing was approximated by a Gamma shadowing leading to the $\mathcal{K}$-distribution [4] and its generalized version [5]. This versatile distribution proved to be particularly useful in evaluating the performance of composite channels [5]-[9]. In the view of the appropriateness of this distribution for characterizing real-world communication links, it is appealing to inspect the capacity of these channels.

Spectral efficiency of adaptive transmission techniques has received extensive interest in the last decade. In [10], the authors examined the capacity of Rayleigh fading channels under different adaptive transmission techniques and different configurations. Other fading channels, like Nakagami, Rician and Weibull fading, were studied in [11] and [12]. In this paper, we provide a thorough analysis of the capacity of generalized-$\mathcal{K}$ fading channels under different adaptive transmission techniques. Moreover, we show that when the shaping parameters of the generalized-$\mathcal{K}$ pdf take certain special values, we can obtain simple expressions for the capacity. Another contribution of this paper is that we provide a closed-form expression for the outage probability when the shaping parameters are integers[1].

Over a generalized-$\mathcal{K}$ fading channel, the pdf of the output signal to noise ratio (SNR) is given by [7]

$$p(\gamma) = \frac{a^{\beta+1}}{2^{\beta}\Gamma(m)\Gamma(k)}\gamma^{\frac{\beta-1}{2}}K_{\alpha}(a\sqrt{\gamma}), \qquad (1)$$

where k and m are the shaping parameters of the distribution, with $\alpha = k - m$ and $\beta = k + m - 1$, and $K_{\alpha}(\cdot)$ is the modified Bessel function of the second kind and order α. In (1), $a = \sqrt{\frac{4km}{\overline{\gamma}}}$ where $\overline{\gamma}$ is the average SNR. In general, the parameter m is a positive real number. But in our analysis, we will assume that m is an integer. On the other hand, we impose no condition on k and assume that this parameter can take arbitrary positive real values. An important special case falls under the assumption that $m = 1$, and this corresponds to the $\mathcal{K}$-distribution. One of the nice properties of this distribution is that the pdf of the instantaneous SNR at the output of a maximum ratio combiner with M i.i.d. branches is readily obtained from (1) by substituting m with Mm and $\overline{\gamma}$ with $M\overline{\gamma}$ [6]. Consequently, all the following study applies also if maximum ratio combining (over i.i.d. fading) is employed at the receiver.

The remainder of the paper is organized as follows. In section II, we study the capacity of optimal rate adaptation with constant transmit power. In section III, we provide closed-form expressions for the capacity with optimal power and rate adaptation. The capacity with channel inversion and fixed rate is then examined in Section IV. In Section V, we give some numerical results and show that the analytical expressions obtained match perfectly the results obtained by computer simulation. Finally, the paper concludes with a summary of the main results in Section VI.

II. Optimal Rate Adaptation with Constant Transmit Power

Under the optimal rate constant power (ora) policy, the capacity is known to be given by [10]

$$< C >_{\text{ora}} = \frac{a^{\beta+1}}{2^{\beta}\Gamma(m)\Gamma(k)}\int_{0}^{+\infty}\log_2(1+\gamma)\gamma^{\frac{\beta-1}{2}}K_{\alpha}(a\sqrt{\gamma})d\gamma. \qquad (2)$$

[1]It should be stressed here that the closed-form expression in terms of the hypergeometric function that was presented in [7] can not be used if α takes integer values, since $csc(\pi\alpha)$ is zero in this case.

For arbitrary α and β, this capacity has been expressed in terms of Meijer-G functions in [7]. We will show next that the capacity can be also written in terms of Lommel functions. Then, since the evaluation of Meijer-G functions can be sometimes laborious, we will show that, when k is an integer plus one half, the capacity can be written in terms of the more familiar sine and cosine integrals.

A. The capacity in terms of Lommel functions

With the change of variable $x = \sqrt{\gamma}$ in (2), the capacity can be re-written as follows

$$< C >_{\text{ora}} = \frac{a^{\beta+1}}{2^{\beta-1}\Gamma(m)\Gamma(k)\ln(2)} A_{\alpha,m}, \tag{3}$$

where $A_{\alpha,m}$ is given by

$$A_{\alpha,m} = \int_0^{+\infty} \ln(1+x^2) x^{\alpha+2m-1} K_\alpha(ax) dx, \tag{4}$$

and is derived in terms of the Lommel Functions in Appendix I. Hence, using this, we obtain the closed-form expression (5) provided at the bottom of this page, where $(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)}$ designates the Pochhammer symbol and $S_{\mu,\nu}(.)$ are the Lommel functions[2].

B. Capacity when k is equal to an integer plus one-half

Assume now that there is an integer n such that $k = n + \frac{1}{2}$. Using [14, Eq.(8.468)], we have[3]

$$K_\alpha(ax) = \sqrt{\frac{\pi}{2ax}} \exp(-ax) \sum_{l=0}^{n-m} \frac{\Gamma(n-m+1+l)(2ax)^{-l}}{\Gamma(n-m+1-l)\Gamma(l+1)}. \tag{6}$$

The equation above, when injected in (3), gives

$$< C >_{\text{ora}} = \frac{a^{\beta+0.5}\sqrt{\pi}\sum_{l=0}^{n-m} \frac{\Gamma(n-m+1+l)L_l(a)}{\Gamma(n-m+1-l)\Gamma(l+1)(2a)^l}}{2^{\beta-0.5}\Gamma(m)\Gamma(k)\ln(2)}, \tag{7}$$

where $L_l(a)$ is given by

$$L_l(a) = \int_0^{+\infty} \ln(1+x^2) x^{n+m-l-1} \exp(-ax) dx, \tag{8}$$

which can be evaluated using successive integration by part and [14, Eq.(2.321.2)] as

$$L_l(a) = 2\sum_{j=0}^{n+m-l-1} \frac{(n+m-l-j)_j}{a^{j+1}} \int_0^{+\infty} \frac{x^{n+m-l-j}}{1+x^2} \exp(-ax) dx. \tag{9}$$

After inserting (9) in (7), we obtain that the capacity with ORA is

$$\begin{aligned} < C >_{\text{ora}} = & \frac{a^{\beta+0.5}\sqrt{\pi}}{2^{\beta-1.5}\Gamma(m)\Gamma(k)\ln(2)} \sum_{l=0}^{n-m} \frac{\Gamma(n-m+1+l)}{\Gamma(n-m+1-l)} \\ & \times \frac{1}{\Gamma(l+1)(2a)^l} \sum_{j=0}^{n+m-l-1} \frac{(n+m-l-j)_j}{a^{j+1}} \Upsilon_{j,l}(a), \end{aligned} \tag{10}$$

where $\Upsilon_{j,l}(a)$ can be evaluated using [14, Eq.(3.356.1)] and [14, Eq.(3.356.2)] and is given in (11) at the bottom of this page. In this expression, $ci(.)$ and $si(.)$ are the cosine and the sine integrals, which are known to be defined by

$$ci(x) = -\int_x^{+\infty} \frac{\cos(t)}{t} dt \quad \text{and} \quad si(x) = -\int_x^{+\infty} \frac{\sin(t)}{t} dt, \tag{12}$$

respectively.

III. Optimal Simultaneous Power and Rate Adaptation

A. Capacity

For optimal power and rate adaptation (opra), the capacity is known to be given by [10]

$$< C >_{\text{opra}} = \frac{a^{\beta+1}\int_{\gamma_0}^{+\infty} \log_2(\frac{\gamma}{\gamma_0})\gamma^{\frac{\beta-1}{2}} K_\alpha(a\sqrt{\gamma}) d\gamma}{2^\beta \Gamma(m)\Gamma(k)} \tag{13}$$

$$= \frac{b^{\beta+1}\int_1^{+\infty} \log_2(x) x^{\alpha+2m-1} K_\alpha(bx) dx}{2^{\beta-2}\Gamma(m)\Gamma(k)}, \tag{14}$$

where $b = a\sqrt{\gamma_0}$. Using partial integration, we can rewrite the last equation as

$$< C >_{\text{opra}} = \frac{-b^{\beta+1}}{2^{\beta-2}\Gamma(m)\Gamma(k)\ln(2)} \underbrace{\int_1^{+\infty} \frac{J_{\alpha,m}(x)}{x} dx}_{-I_{\alpha,m}}, \tag{15}$$

[2]For a thorough presentation of the Lommel functions, we refer the interested reader to [15].

[3]Since $K_{-\nu}(x) = K_\nu(x)$, we assume for convenience and without loss of generality that $n \geq m$.

$$\begin{aligned} < C >_{\text{ora}} = & \frac{a^{\beta+1}}{2^{\beta-1}\Gamma(m)\Gamma(k)\ln(2)} \sum_{j=1}^{m} \left(\frac{2}{a}\right)^{j-1} (-1)^{m-j}(m-j+1)_{j-1} 2^{\alpha+j+1}\Gamma(\alpha+j+1) \frac{S_{-1-\alpha-j,\alpha+j}(a)}{a} \\ & - \frac{a^{\beta+1}}{2^{\beta-1}\Gamma(m)\Gamma(k)\ln(2)} \sum_{j=1}^{m-1} \left(\frac{2}{a}\right)^{j-1} (m-j+1)_{j-1} \sum_{l=1}^{m-j} \frac{(-1)^{m-j-1-l}\Gamma(\alpha+l+j) 2^{2l+\alpha+j-1}(l-1)!}{a^{2l+1+\alpha+j}}, \end{aligned} \tag{5}$$

$$\Upsilon_{j,l}(a) = \begin{cases} (-1)^{q-1}[ci(a)\cos(a) + si(a)\sin(a)] + \frac{1}{a^{2q}}\sum_{t=1}^{q}(2q-2t+1)!(-1)^{t-1}a^{2t-2}, \text{if } n+m-l-j = 2q+1, \\ (-1)^q[ci(a)\sin(a) - si(a)\cos(a)] + \frac{1}{a^{2q-1}}\sum_{t=1}^{q}(2q-2t)!(-1)^{t-1}a^{2t-2}, \text{if } n+m-l-j = 2q, \end{cases} \tag{11}$$

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where $J_{\alpha,m} = \int x^{\alpha+2m-1} K_\alpha(bx)dx$ is evaluated in Appendix II. Hence, using this, we obtain

$$\begin{aligned} I_{\alpha,m} &= \frac{2^{m-1}(m-1)!}{b^m}\int_1^{+\infty} x^{k-1}K_k(bx)dx + \sum_{l=1}^{m-1}\frac{2^{l-1}}{b^l} \\ &\times (m-l+1)_{l-1}\underbrace{\int_1^{+\infty} x^{\alpha+2m-l-1}K_{\alpha+l}(bx)dx}_{-J_{\alpha+l,m-l}(1)} \quad (16) \\ &= \frac{2^{m-1}(m-1)!}{b^m}I_k + \sum_{l=1}^{m-1}\frac{2^{l-1}(m-l+1)_{l-1}}{b^l} \\ &\times \sum_{j=1}^{m-l}\frac{2^{j-1}(m-l-j+1)_{j-1}}{b^j}K_{\alpha+l+j}(b), \quad (17) \end{aligned}$$

where $I_k = \int_1^{+\infty} x^{k-1}K_k(bx)dx$. Depending on the value of k, we will distinguish three cases: arbitrary k values, integer values of k, and k equal to an integer plus one half.

1) Capacity for arbitrary k: Using [14, Eq.(6.592.4)] and some simplifications, I_k will be written in terms of the Meijer G-function as follows

$$I_k = 2^{k-2}b^{-k}G_{1,3}^{3,0}\left(\frac{b^2}{4}\,\middle|\,\begin{matrix}1\\0,k,0\end{matrix}\right), \quad (18)$$

where, in the last equality, we have used [16, Eq.(07.34.16.0001.01)]. This last formula is valid for any value of k. However, we will see in the next section that, for some specific values of k, it is possible to obtain closed-form expressions in terms of more conventional functions.

2) Capacity for integer values of k: We evaluate I_k by using the following recursion formula:

$$K_k(bx) = K_{k-2}(bx) + 2\frac{(k-1)}{bx}K_{k-1}(bx). \quad (19)$$

By multiplying the last equation by x^{k-1} and integrating, we obtain:

$$\begin{aligned} I_k &= \int_1^{+\infty} x^{k-1}K_{k-2}(bx)dx + 2\frac{(k-1)}{b}I_{k-1} \quad (20) \\ &= \frac{K_{k-1}(b)}{b} + 2\frac{(k-1)}{b}I_{k-1}. \quad (21) \end{aligned}$$

Iterating on this equation and using the fact that $\int K_1(x)dx = -K_0(x)$, we obtain

$$I_k = (k-1)!\sum_{j=0}^{k-1}\frac{1}{j!}\left(\frac{2}{b}\right)^{k-1-j}\frac{K_j(b)}{b}. \quad (22)$$

3) Capacity when k is equal to an integer plus one-half: Assume now that there is an integer n such that $k = n + \frac{1}{2}$. By injecting (6) in I_k, we obtain the following closed-form expression:

$$I_k = \sqrt{\frac{\pi}{2b}}\frac{1}{b^n}\sum_{l=0}^{n}\frac{\Gamma(n+1+l)}{\Gamma(n+1-l)\Gamma(l+1)2^l}\Gamma(n-l,b), \quad (23)$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function.

B. Outage probability

Using the expression of $J_{\alpha,m}$ developed in Appendix II, the outage probability[4] can be written as

$$P_{\text{out}} = 1 - \frac{b^{\beta+1}\sum_{l=1}^{m}\frac{2^{l-1}(m-l+1)_{l-1}K_{\alpha+l}(b)}{b^l}}{2^{\beta-1}\Gamma(m)\Gamma(k)}. \quad (24)$$

C. Optimal cutoff

The optimal cutoff satisfies [10]

$$\int_{\gamma_0}^{+\infty}\left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right)p_\gamma(\gamma)d\gamma = 1, \quad (25)$$

after some manipulations it is found that γ_0 is a solution of the following equation

$$\frac{b^{\beta+1}}{2^{\beta-1}\Gamma(m)\Gamma(k)}\left(-J_{\alpha,m}(1) + J_{\alpha,m-1}(1)\right) - \gamma_0 = 0. \quad (26)$$

For $m \geq 2$, $J_{\alpha,m-1}(1)$ is computed using (40), and, for $m = 1$, we have that $J_{\alpha,0}(1) = I_\alpha$.

IV. Channel Inversion with Fixed Rate

A. Total channel inversion

The capacity for channel inversion with fixed rate (cifr) is, as shown in [10],

$$< C >_{\text{cifr}} = \log_2\left(1 + \frac{2^{\beta-1}\Gamma(m)\Gamma(k)}{a^{\beta+1}\int_0^{+\infty} x^{\beta-2}K_\alpha(ax)dx}\right). \quad (27)$$

We can show that, if $m \leq \frac{3}{2}$ (i.e., $m = 1$ since we consider only integer values), the integral diverges, and, therefore the capacity for channel inversion with fixed rate is zero. Now, if $m \geq 2$, we have that[5]

$$< C >_{\text{cifr}} = \frac{1}{\ln(2)}\ln\left(1 - \frac{2^{\beta-1}\Gamma(m)\Gamma(k)}{a^{\beta+1}\lim_{x\to 0}J_{\alpha,m-1}(x)}\right). \quad (28)$$

Using (37), we obtain the following capacity expression

$$< C >_{\text{cifr}} = \frac{1}{\ln(2)}\ln\left(1 + \overline{\gamma}\frac{(m-1)(k-1)}{mk}\right). \quad (29)$$

Note that when m and k tend to infinity, we approach the capacity of the AWGN channel.

B. Truncated channel inversion

The capacity of truncated channel inversion with fixed rate (tcifr) is given by [10]

$$< C >_{\text{tcifr}} = \log_2\left(1 - \frac{2^{\beta-1}\Gamma(m)\Gamma(k)\gamma_0}{b^{\beta+1}J_{\alpha,m-1}(1)}\right)(1 - P_{\text{out}}). \quad (30)$$

For $m \geq 2$, $J_{\alpha,m-1}(1)$ is computed using (40), and, for $m = 1$, we have $J_{\alpha,0}(1) = I_\alpha$.

[4]The cumulative distribution function (CDF) of a generalized-$\mathcal{K}$ random variable (and a fortiori the outage probability) was given in [7] in terms of the hypergeometric function. However, the provided CDF can not be used if α is an integer.

[5]Here $J_{\alpha,m-1}(x)$ refers to the same function as in (40), but we replace b by a.

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V. Numerical Results

The following analysis is conducted in two different shadowing scenarios that correspond to Loo's model (refer to [3] and the references therein), namely, infrequent light shadowing and frequent heavy shadowing. The corresponding standard deviations σ of the log-normal shadowing are equal, respectively, to 0.115 and to 0.806. By a moment matching technique [8], the parameter k of the generalized-$\mathcal{K}$ distribution can be linked to σ as $k = \frac{1}{e^{\sigma^2}-1}$, which translates therefore to the following values for k: 75.1155 and 1.0931. Throughout our simulations, the parameter m is arbitrarily set to 2 and the diversity order M for MRC was set to 4. The performance in a Nakagami-m channel [13] with $m = 2$ is also provided as a reference.

Figs. 1, 2 and 3 show the capacity of the different policies as well as the outage probability. Note the concordance between the capacity given by the theoretical formulas and the one obtained by computer simulations. As expected, the performance deteriorates as the shadowing becomes more pronounced and the results in the light shadowing conditions are almost the same as in the Nakagami-m channel. Fig. 1 shows also that, compared to optimal power and rate adaptation, transmission with optimal rate adaptation suffers capacity penalty at low SNR only. However, as $\overline{\gamma}$ increases, the two policies will provide the same capacity. In the light shadowing conditions, as illustrated in Fig. 2, this comparison holds also for truncated channel inversion and total channel inversion. However, for heavy shadowing, total channel inversion exhibits large capacity loss.

VI. Conclusion

In this paper we have presented several results for the capacity of generalized-$\mathcal{K}$ fading channels. More specifically, we have obtained closed-form expressions for the capacity of three adaptive schemes, namely, i) optimal rate adaptation with constant power, ii) optimal rate and power adaptation and iii) channel inversion with fixed rate. Comparisons with numerical simulations showed the accuracy of our proposed formulas.

Appendix I
Evaluation of $A_{\alpha,m}$

Relying on partial integration and using [14, Eq. (2.732)][6]

$$\int x^{2n+1}\ln(1+x^2)dx = \frac{1}{2n+2}\left((x^{2n+2}+(-1)^n)\ln(1+x^2) + \sum_{l=1}^{n+1}\frac{(-1)^{n-l}}{l}x^{2l}\right), \quad (31)$$

[6]There is a typo in Eq.(2.732): $\frac{1}{2n+1}$ should be replaced by $\frac{1}{2n+2}$.

as well as the fact that $\frac{dx^{\nu}K_{\nu}(ax)}{dx} = -ax^{\nu}K_{\nu-1}(ax)$, we prove that $A_{\alpha,m}$ satisfies the following recursion formula:

$$\begin{aligned} A_{\alpha,m} &= \frac{2(m-1)}{a}A_{\alpha+1,m-1} + (-1)^{m-1}A_{\alpha,1} \\ &+ \sum_{l=1}^{m-1}\frac{(-1)^{m-2-l}}{l}\lim_{x\to 0}J_{\alpha,l+1}(x), \end{aligned} \quad (32)$$

where we have used the fact that

$$\int_0^{+\infty} x^{2l+\alpha+1}K_{\alpha}(ax)dx = -\lim_{x\to 0}J_{\alpha,l+1}(x), \quad (33)$$

and $J_{\alpha,l}(x)$ is derived in the next appendix. Iterating over this equation, we obtain that

$$A_{\alpha,m} = \sum_{j=1}^{m}\left(\frac{2}{a}\right)^{j-1}(-1)^{m-j}(m-j+1)_{j-1}A_{\alpha+j-1,1} + \sum_{j=1}^{m-1}\left(\frac{2}{a}\right)^{j-1} (m-j+1)_{j-1}\sum_{l=1}^{m-j}\frac{(-1)^{m-j-1-l}\lim_{x\to 0}J_{\alpha+j-1,l+1}(x)}{l}. \quad (34)$$

Using partial integration along the fact that $\int u^{\zeta+1}K_{\zeta}(au)du = -u^{\zeta+1}K_{\zeta+1}(au)/a$, we obtain that $A_{\zeta,1}$ is given by:

$$A_{\zeta,1} = \frac{2}{a}\int_0^{+\infty}\frac{x^{\zeta+2}}{1+x^2}K_{\zeta+1}(ax)dx. \quad (35)$$

Using [14, Eq.(6.565.7)], we obtain the following closed-form expression for $A_{\zeta,1}$:

$$A_{\zeta,1} = 2^{\zeta+2}\Gamma(\zeta+2)\frac{S_{-2-\zeta,\zeta+1}(a)}{a}, \quad (36)$$

where $S_{\mu,\nu}(.)$ is the Lommel function.
Using the fact that $K_t(x) \underset{x\to 0}{\sim} \frac{\Gamma(t)}{2}\left(\frac{2}{x}\right)^t$ along with the result of Appendix II, we prove that

$$\lim_{x\to 0}J_{\alpha+j-1,l+1}(x) = -\frac{\Gamma(\alpha+l+j)2^{2l+\alpha+j-1}l!}{a^{2l+1+\alpha+j}}. \quad (37)$$

Finally, $A_{\alpha,m}$ will be given by (38) at the bottom of this page.

Appendix II
Evaluation of $J_{\xi,p}$

Let $J_{\xi,p}(x) = \int x^{\xi+2p-1}K_{\xi}(bx)dx$. Using partial integration along the fact that $\int x^{\xi+1}K_{\xi}(bx)dx = -\frac{x^{\xi+1}K_{\xi+1}(bx)}{b}$, we obtain

$$J_{\xi,p}(x) = \frac{2p-2}{b}J_{\xi+1,p-1}(x) - \frac{x^{\xi+2p-1}K_{\xi+1}(bx)}{b}. \quad (39)$$

Iterating on this equation and using the fact that $J_{\xi+p-1,1}(x) = -\frac{x^{\xi+p}K_{\xi+p}(bx)}{b}$, we obtain

$$J_{\xi,p}(x) = -\sum_{l=1}^{p}\frac{2^{l-1}(p-l+1)_{l-1}}{b^l}x^{\xi+2p-l}K_{\xi+l}(bx). \quad (40)$$

$$\begin{aligned} A_{\alpha,m} &= \sum_{j=1}^{m}\left(\frac{2}{a}\right)^{j-1}(-1)^{m-j}(m-j+1)_{j-1}2^{\alpha+j+1}\Gamma(\alpha+j+1)\frac{S_{-1-\alpha-j,\alpha+j}(a)}{a} \\ &- \sum_{j=1}^{m-1}\left(\frac{2}{a}\right)^{j-1}(m-j+1)_{j-1}\sum_{l=1}^{m-j}\frac{(-1)^{m-j-1-l}\Gamma(\alpha+l+j)2^{2l+\alpha+j-1}(l-1)!}{a^{2l+1+\alpha+j}}. \end{aligned} \quad (38)$$

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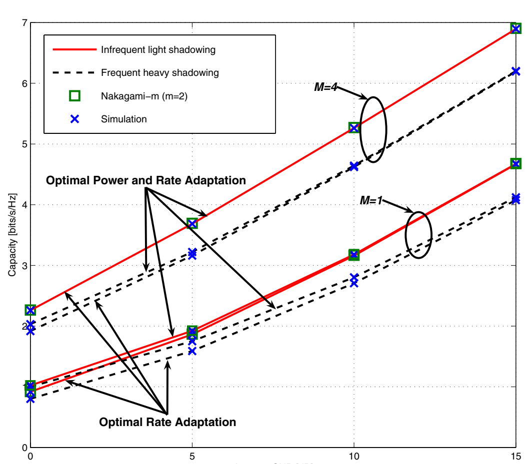

Fig. 1. Capacity for ORA and OPRA policies versus SNR in infrequent light shadowing ($k = 75.1155$) and frequent heavy shadowing ($k = 1.0931$) environments.

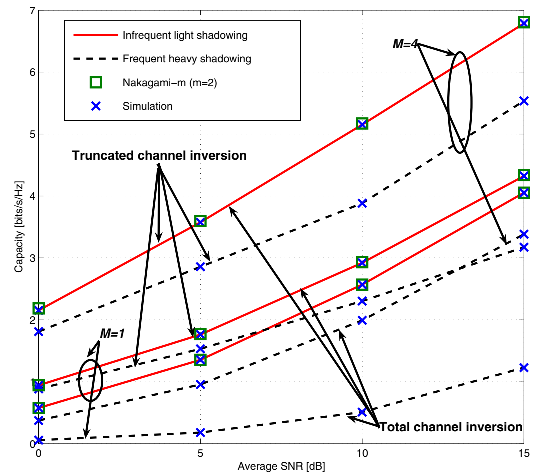

Fig. 2. Capacity for total channel inversion and truncated channel inversion policies versus SNR in infrequent light shadowing ($k = 75.1155$) and frequent heavy shadowing ($k = 1.0931$) environments.

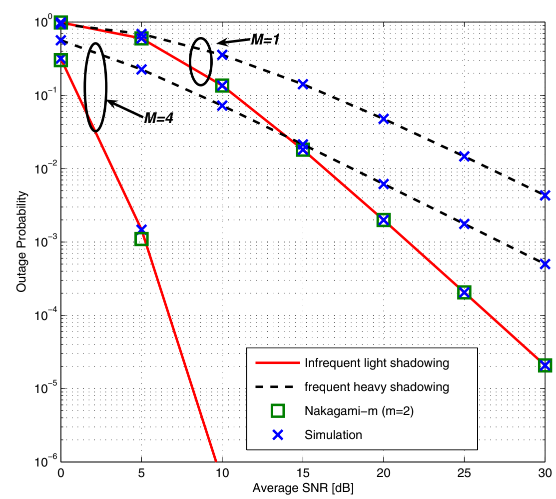

Fig. 3. Outage probability for $\gamma_0 = 5$ dB in infrequent light shadowing ($k = 75.1155$) and frequent heavy shadowing ($k = 1.0931$) environments.