SIR Optimized Hermite-Based Pulses for BFDM Systems in Doubly Dispersive Channels

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***Abstract*— This paper deals with performance evaluation of an optimized Pulse Shaping BFDM system operating in doubly dispersive channels. An SIR-based optimization is done by searching appropriate pulse shape according to the known channel statistics. We show that, in doubly dispersive channels, the impact of interference, composed by both inter-symbol interference (ISI) and inter-carrier interference (ICI), depends crucially on time frequency localization of transmit and receive pulses. We propose to maximize the SIR, written as a ratio of two quaternary forms, by alternating between transmit and receive pulse spaces both generated by Hermite Waveforms. Numerical results involve PS-BFDM performance evaluation in brick with constant dispersion and Jakes shaped channel, and yet a performance comparison with a state of the art CP-OFDM system.**

***Index Terms*— Pulse shaping, time-frequency localization, Hermite waveforms, doubly dispersive channels.**

I. Introduction

In conventional orthogonal frequency division multiplexing (OFDM) systems, a high bit rate data stream is divided into several low bit rate data substreams each modulated over a time-domain rectangular shaped sub-carrier. Even if this technique ensures a reduced amount of ISI in weakly dispersive channels, it is still vulnerable to highly time dispersive channels and useless in doubly dispersive channel where the Doppler effect destroys the orthogonality between subcarriers and causes ICI/ISI. So far, different techniques have been proposed to suppress ISI/ICI. The ISI can be suppressed by cyclically extending a rectangular-shaped symbol with a cyclic prefix (CP-OFDM) [1]. Nevertheless, the used rectangular pulse shape is not adapted to highly frequency dispersive channels due to its poor frequency localization proprieties. In pulse shaping BFDM [2], a smoother and more localized pulse is used to replace the rectangular one. While pulse shaping OFDM was proposed rather early [3], only recently the design of the BFDM transmit and receive pulses has been considered in more detail [4,5]. By considering biorthogonality instead of orthogonality, more localized pulses can be designed for increased channel dispersion robustness and system performance improvement. In [6], the duality of multicarrier systems and Weyl-Heisenberg (or Gabor) frames is elaborated and applied to the design of OFDM and BFDM systems. In this paper, we present an exact ISI/ICI analysis of an BFDM system operating over a time frequency selective channel. Next, we propose a pulse optimization method that performs an explicit maximization of the SIR. An optimum pulse is formed by linearly combining Hermite waveforms. We then, present a comparative performance evaluation of our proposed system and a classical CP-OFDM system over a wireless channel with reasonably realistic statistics.

This paper is organized as fellows: After presenting the BFDM system model in section II, the theoretical computation of biorthogonal pulses is discussed in section III. There, a SIR formulation and methods maximizing it are provided. In section IV, performance of the PS-BFDM system is assessed in two differently shaped doubly dispersive channels. A performance comparison between the PS-BFDM system and CP-OFDM system is provided. Finally, a conclusion is provided in section V.

II. System Model

We consider a pulse-shaping OFDM/BFDM system deploying a time frequency rectangular lattice. Data symbols are transmitted on the lattice grid points after being modulated by carrier waveforms given by:

$$g_{lm}(t) = g(t - lT)e^{j2\pi mFt}, \tag{1}$$

which are time-frequency shifted versions of the normalized energy prototype pulse $g(t)$. Note that T is the OFDM symbol duration and F is the sub-carriers frequency spacing. The baseband transmit signal is given by:

$$x(t) = \sum_{l=-\infty}^{\infty} \sum_{m=0}^{N_c-1} X_{lm} g(t - lT)e^{j2\pi mFt}, \tag{2}$$

where X_{lm} is a complex data symbol transmitted at time lT and sub-carrier frequency mF. The wireless channel is modeled as a random linear time varying system H. The channel output (received signal) is thus given by:

$$y(t) = H(x(t)) = \int_\nu \int_\tau h(\tau, \nu)x(t - \tau)e^{j2\pi\nu t} d\tau d\nu, \tag{3}$$

where $h(\tau, \nu)$ is the delay-Doppler spread function witch is a random process in (τ, ν) [7]. The statistical description of the channel is greatly simplified by the wide-sense stationary scattering (WSSUS) assumption [7]:

$$\begin{aligned} &E\{h(\tau, \nu)\} = 0, \\ &E\{h(\tau, \nu)h^*(\tau_1, \nu_1)\} = S(\tau, \nu)\delta(\tau - \tau_1)\delta(\nu - \nu_1), \end{aligned} \tag{4}$$

where $S(\tau, \nu)$ is the channel dispersion function assumed to be supported within the rectangle $[0, T_m] \times [-B_d/2, B_d/2]$,

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where T_m and B_d are, respectively, the Delay channel spread and Doppler channel spread.

In this paper, we chose not to focus on the additive noise component because we are not interested in noise effects. At the receiver, the demodulator computes, for each data symbol X_{lm}, a decision variable Y_{lm} by calculating the inner product given by:

$$y_{lm} = \int y(t) f_{lm}^*(t) dt, \tag{5}$$

where $y(t)$ is the received signal and $f_{lm}(t)$ is a time-frequency shifted version of the normalized energy receive pulse $f(t)$, defined as:

$$f_{lm}(t) = f(t - lT) e^{j2\pi mFt}. \tag{6}$$

In the special case where $f(t) = g(t)$, we deal with a pulse shaping OFDM system.

In fact, we assume a centered channel since we do not delay the receive impulse by the mean channel delay $T_m/2$. Hence:

$$y_{lm} = \sum_{l_1 m_1} X_{l_1 m_1} h_{lml_1m_1}(t), \tag{7}$$

where $h_{lml_1m_1}$ represents the global channel given by:

$$h_{lml_1m_1} = \int\int h(\tau,\nu) A_{fg}^*((l_1 - l)T + \tau, (m_1 - m)F + \nu) \\ e^{-j2\pi m_1 F\tau} e^{j2\pi lT\nu} e^{-j2\pi lFT(m-m_1)} d\tau d\nu, \tag{8}$$

where A_{fg} is the cross ambiguity function between $f(t)$ and $g(t)$ defined as:

$$A_{fg}(\phi,\psi) = \int f(t) g^*(t-\phi) e^{-j2\pi\psi t} dt. \tag{9}$$

III. MC PULSES AND PERFORMANCE ANALYSIS

A. SIR formulation

The decision variable corresponding to the demodulating symbols X_{lm} is expressed as:

$$Y_{lm} = X_{lm} h_{lmlm} + \sum_{(l_1,m_1)\neq(l,m)} X_{l_1m_1} h_{lml_1m_1}. \tag{10}$$

The mean ISI/ICI power is defined as:

$$\sigma_I^2 = E\{\mid Y_{lm} - X_{lm} h_{lmlm} \mid^2\}, \tag{11}$$

whereas the mean power of the desired component is defined as:

$$\sigma_I^2 = E\{\mid X_{lm} h_{lmlm} \mid^2\}. \tag{12}$$

We also assume that X_{lm} are independent identically distributed (iid) data symbols with zero mean and average transmit energy E_s . We can then show that the ISI/ICI power is given by:

$$\sigma_I^2 = E_s \sum_{(l,m)\neq(0,0)} \int\int S(\tau,\nu) |A_{fg}(lT+\tau, mF+\nu)|^2 d\tau d\nu. \tag{13}$$

Similarly, the mean power of the desired component can be expressed as:

$$\sigma_S^2 = E_s \int\int S(\tau,\nu) |A_{fg}(\tau,\nu)|^2 d\tau d\nu. \tag{14}$$

Then, we define the SIR as:

$$SIR = \frac{\sigma_S^2}{\sigma_I^2}. \tag{15}$$

According to (13), for the ISI/ICI to be low, $|A_{fg}(\tau,\nu)|^2$ must be small within all time-frequency plane regions of the form $R_{lm} = [lT - T_m/2, lT + T_m/2] \times [mF - B_d/2, mF + B_d/2], (l,m) \neq (0,0)$. Such a behavior is favored by weakly dispersive channels (small delay spread T_m and Doppler spread B_d) and small time-frequency lattice densities (large time-frequency spacing product TF). Unfortunately, channel characteristics are generally out of control. While small lattice densities mean a poor modulation spectrum efficiency, larger values of TF increase the freedom in designing pulses $g(t)$ and $f(t)$ satisfying biorthogonality property. Hence, for an MC system, low ISI/ICI is favored by the use of well localized pulses and the choice of lattice density corresponds to a trade-off between spectral efficiency and freedom in the system design.

B. SIR optimized Hermite-based pulses

Hermite waveforms form an orthonormal base of the Hilbertian space of square integrable functions and provide, in decreasing order, maximum localization in time and frequency. Hence we will form all candidate pulses as a linear combination of Hermite waveforms:

$$g(t) = \sum_k \alpha_k u_k(t) \text{ and } f(t) = \sum_p \beta_p u_p(t). \tag{16}$$

As noticed in [8] from simulation results, the optimum pulse coefficients α_k, and similarly β_k, decrease exponentially with respect to k. Hence, for complexity reasons, we truncate the representation of candidate pulses to the N most concentrated Hermite waveforms. The optimization task amounts, then, to finding $\{\alpha_k\}_{k=0}^{k=N-1}$ and $\{\beta_k\}_{k=0}^{k=N-1}$ so that the target function $SIR = \sigma_s^2/\sigma_I^2$ is maximized. Using the pulse expressions below, we can formulate the cross-ambiguity function as:

$$A_{fg}(\tau,\nu) = \sum_{k=0}^{N-1}\sum_{p=0}^{N-1} \alpha_k^* \beta_p A_{kp}(\tau,\nu), \tag{17}$$

where A_{kp} is the cross ambiguity of $u_k(t)$ and $u_p(t)$ [8]:

$$A_{kp} = \int u_p(t) u_k^*(t-\tau) e^{-j2\pi\nu t} dt. \tag{18}$$

A_{kp} can be expressed as:

$$A_{kp}(\tau,\nu) = (-1)^{k+p} \sqrt{\tfrac{p!}{k!}} e^{-\frac{1}{2}(\tau^2+\nu^2)} (\sqrt{\pi})^{k-p} \\ (\tau + i\nu)^{k-p} L_p^{(k-p)}(\pi(\tau^2+\nu^2)), \tag{19}$$

where L is the generalized Laguerre polynomial.

Based on that, the signal power becomes:

$$\sigma_S^2 = E_s \sum_{k,p\leq N-1}\sum_{k_1,p_1\leq N-1} \alpha_k^* \beta_p \alpha_{k_1} \beta_{p_1}^* D(k,p,k_1,p_1), \tag{20}$$

where:

$$D(k,p,k_1,p_1) = \int\int S(\tau,\nu) A_{kp}(\tau,\nu) A_{k_1p_1}^*(\tau,\nu) d\tau d\nu. \tag{21}$$

Similarly, the mean interference power is written as:

$$\sigma_I^2 = E_s \sum_{k,p\leq N-1}\sum_{k_1,p_1\leq N-1} \alpha_k^* \beta_p \alpha_{k_1} \beta_{p_1}^* I(k,p,k_1,p_1), \tag{22}$$

where :

$$I(k,p,k_1,p_1) = \int\int S(\tau,\nu)\sum_{(l,m)\neq(0,0)} A_{kp}(lT+\tau,mF+\nu)A^*_{k_1p_1}(lT+\tau,mF+\nu)d\tau d\nu. \quad (23)$$

Let $U^T = [\alpha_0...\alpha_{N-1}]$ and $V^T = [\beta_0...\beta_{N-1}]$, then the SIR becomes:

$$SIR = \frac{(U\otimes V^*)^H D(U\otimes V^*)}{(U\otimes V^*)^H I(U\otimes V^*)}, \quad (24)$$

where $\otimes$ is the Kronecker tensor product, D is a $(N^2\times N^2)$ hermitian matrix of elements $D(k,p,k_1,p_1)$ and I is a $(N^2\times N^2)$ hermitian positive matrix of elements $I(k,p,k_1,p_1)$. Formulated as below, the SIR is the target function to be maximized so that optimal pulses are reached.

C. Optimization of biorthogonal receive and transmit pulses (PS-BFDM)

In a biorthogonal system, we are interested in finding the transmit and receive pulses $g(t)$ and $f(t)$. The optimization procedure is based on alternating between transmit and receive Hermite generated spaces by iteratively and alternatively fixing the transmit and the receive pulse to optimize the other. By fixing either U or V, the SIR becomes a ratio of two quadratic forms easy to optimize. In the initialization step, we begin by setting V or U at a random vector. In the i-th iteration, the optimal achievable SIR corresponds to the maximum generalized eigenvalue of the pair of symmetric matrices $(D_1^{(i)}, I_1^{(i)})$ given by:

$$\begin{aligned}\lambda_{max}(D_1^{(i)}, I_1^{(i)}) &= \sup \frac{U^H D_1^{(i)} U}{U^H I_1^{(i)} U} \\ &= \sup\{\lambda / det(\lambda I_1^{(i)} - D_1^{(i)}) = 0\},\end{aligned} \quad (25)$$

where $D_1^{(i)}$ (respectively $I_1^{(i)}$) is the entry-wise product of D (respectively I) by $V^{(i)^*}V^{(i)^T}$. Since transmit and receive pulses have symmetric roles, we always set $V^{(i+1)}$ equal to the obtained vector U in the i-th iteration.

IV. Numerical Results

We consider a pulse shaping OFDM/BFDM system using a time frequency lattice with $TF \succ 1$, transmitting over a WSSUS channel. For simplicity , we consider a constant square-shaped channel scattering function with $T_m = B_d = \sqrt{\beta}$, where β is the channel spread factor. Moreover, for evident symmetry reasons with respect to Doppler and delay spread characteristics, we assume the time-frequency lattice to be square-shaped, with equal carrier separation and symbol period $T = F = \sqrt{TF}$. Due to the $\pi/2$ rotational invariance of the rectangular lattice, the most localized prototype function should be searched among all orthonormal prototype functions with $pi/2$ rotational invariant ambiguity functions [9].By examining the expression of $A_{kp}(\tau,\nu)$, we see that rotational invariance is guaranteed whenever $p-k$ modulo 4 is 0. Then, both p and k can be written as $4c+l, l=0,1,2,3,$ and c is any non-negative integer. Since we want to keep the most localized Gaussian function in the representation of all candidate pulses, we should choose $l=0$. Hence, we express $g(t)$, and similarly $f(t)$ as:

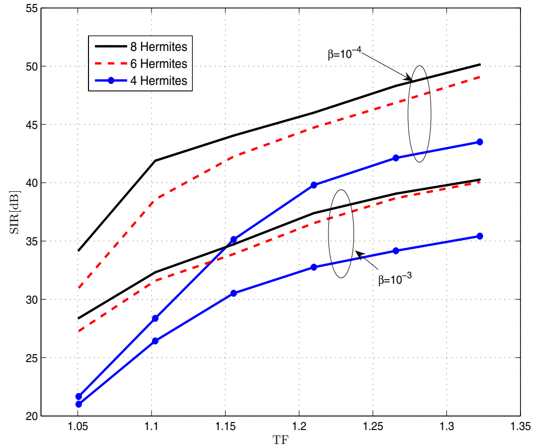

Fig. 1. PS-BFDM optimized SIR versus TF with increased number of Hermite waveforms. .

$$g(t) = \sum_k \alpha_{4k}u_{4k}(t) \text{ and } f(t) = \sum_p \beta_{4p}u_{4p}(t). \quad (26)$$

In figure 1, we show that BFDM system performance in terms of SIR is greatly improved while increasing the number N of Hermite waveforms. Nevertheless, this SIR improvement decreases with N. Adding to that the fact that increasing N induces computational complexity and even causes optimization algorithm instability, we choose $N = 8$ as an optimal size offering good SIR level and acceptable complexity. We are interested in extending our work to a more realistic channel having a classic Doppler power spectrum, known as Jakes model [7], and an exponential multipath power spectrum. To comply with the requirement of centered prototype functions used below, we use a δ_τ delayed version of the dispersion function. The normalized dispersion function is given by:

$$S(\tau,\nu) = S(\tau)S(\nu) \quad (27)$$

with

$$S(\tau) = \begin{cases} \frac{1}{T_m}e^{-\frac{(\tau-\delta_\tau)}{T_m}} & if\ \tau \geq 0 \\ ,0 & \text{otherwise,}\end{cases} \quad (28)$$

$$S(\nu) = \begin{cases} \frac{2}{\pi B_d\sqrt{1-(\frac{2\nu}{B_d})^2}} & if\ -B_d/2 < \nu < B_d/2 \\ ,0 & \text{otherwise.}\end{cases} \quad (29)$$

To adapt our system configuration fixed below, i.e. $T = F$, to this channel, the term $\frac{B_d}{T_m}$ must be chosen so that time and frequency variations occur with the same statiatics, i.e., delays variance is equal to Doppler frequency variance. Since,

$$\begin{aligned} var(\tau) &= \overline{\tau^2} - \overline{\tau}^2 \\ &= \frac{\int \tau^2 S(\tau)d\tau - (\int \tau S(\tau)d\tau)^2}{||S(\tau)||^2} \\ &= T_m^2 \end{aligned} \quad (30)$$

and

$$\begin{aligned} var(\nu) &= \overline{\nu^2} - \overline{\nu}^2 \\ &= \frac{\int \nu^2 S(\nu)d\tau - (\int \nu S(\nu)d\nu)^2}{||S(\nu)||^2} \\ &= \frac{B_d^2}{8} \end{aligned} \quad (31)$$

consequently,

$$B_d = 2\sqrt{2}T_m. \quad (32)$$

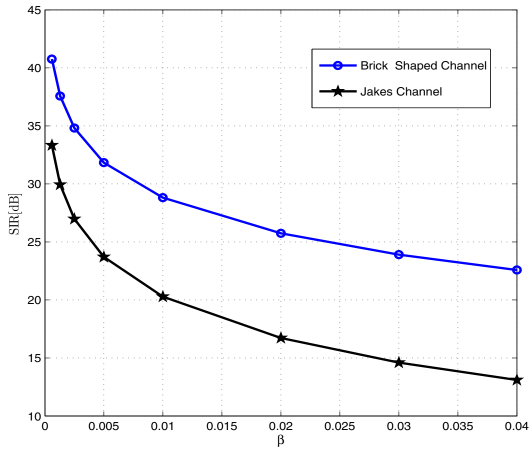

Fig. 2. PS-BFDM performances over two different channel types.

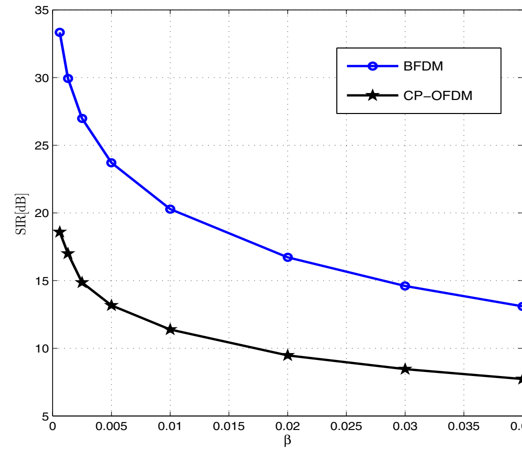

Fig. 3. PS-BFDM/CP-OFDM performance comparison.

In figure 2, the optimized PS-BFDM system performs differently when changing the channel dispersion function shape. We observe to a degradation of almost 10 dB when passing from brick constant dispersion to Jakes shaped channel, mainly for a highly dispersive one. This degradation is due to the spreading of the Jakes shaped channel dispersion function. Now we are interested in comparing an optimized PS-BFDM system to a conventional state-of-the-art CP-OFDM system both operating over a Jakes shaped channel. First we provide the SIR expression in a CP-OFDM system. To do so, let us consider an OFDM system employing a cyclic prefix [1]. Here, $T = 1/F + T_g$, where T_g denotes the length of the CP-OFDM modulator and the demodulator deploy rectangular respectively transmit and receive pulses given by:

$$g(t) = rect_T(t) \text{ and } f(t) = rect_{T-T_g}(t). \tag{33}$$

To compute the SIR expression over Jakes channel model, we must distinguish between $\delta_\tau \succeq -Tg/2$ and $\delta_\tau \preceq -Tg/2$. For $\delta_\tau \succeq -Tg/2$, interference comes only from symbols transmitted before the current one. Thus, interference is only computed by integrating the sum of all time-frequency delayed versions of the square absolute value of the ambiguity function (9) in the plane $\tau \preceq 0$ over the dispersion function support. Since, $\sum_m \mid A_{fg}(\tau, \nu - mF) \mid^2$ is independent of ν, we can easily compute the interference power as:

$$I = T^2(1 - e^{-(T+T_g/2-\delta_\tau)/T_m}) - Te^{-(T_g/2-\delta_\tau)/T_m} (T_m - (T+T_m)e^{-T/T_m}) + TT_m e^{\delta_\tau/T_m} e^{-T/(2T_m)} (e^{(T+T_g)/2T_m} + e^{-(T+T_g/2)/T_m}). \tag{34}$$

We also compute the useful power by evaluating the non delayed contribution of dispersion function:

$$S = \int\int [T^2 sinc^2(\nu T) + (T - \tau + T_g/2)^2 sinc^2(\nu(T - \tau + T_g/2))] S(\tau, \nu) d\tau d\nu. \tag{35}$$

In figure 4, we compare our optimized PS-BFDM design with a conventional state-of-the-art CP-OFDM system which are two biorthogonal modulations deploying, respectively, well and poor frequency localized pulses. We show the SIR obtained with the two systems as a function channel dispersion. Note that, in this simulation, δ_τ was fixed at $-T_g/2$. The CP-OFDM system deploys a cyclic prefix (CP) of length $T_g = 0.25T$ which corresponds to $TF = 1.25$ for both systems. A BFDM system outperforms a CP-OFDM system for a large range of channel dispersion, thanks to the use of well frequency localized pulses.

V. Conclusion

An SIR-based optimized PS-BFDM system was designed and performance evaluation was assessed over both brick shaped constant dispersion channel and Jakes model channel. Simulations show that over doubly dispersive channel, a PS-BFDM system achieves lower interference than a CP-OFDM system, because pulse shaping avoids the poor spectral concentration of the rectangular pulse employed by CP-OFDM. To leverage this advantage, it is necessary to design well time-frequency localized transmit and receive pulses in accordance with the statistical properties of the wireless channel.

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