NEW CLOSED-FORM ESTIMATORS FOR THE ANGLE OF ARRIVAL AND THE ANGULAR SPREAD OF A LOCALLY SCATTERED SOURCE

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ABSTRACT

We estimate the angular spread (AS) and the nominal angle of arrival (AoA) of a locally scattered source using a uniform linear array (ULA) of sensors. First, we use Taylor series expansions to transform this problem into the localization of two point sources as it has been proposed in the literature. Based on the resulting approximate form of the covariance matrix, we directly retrieve analytical expressions for the AS and the nominal AoA. Compared with earlier works, the proposed method does not require the knowledge of the angular distribution of the scattered source. Furthermore, it accurately determines the required parameters in a computationally very simple manner as illustrated by simulations.

1. INTRODUCTION

Local scattering models have recently attracted an increasing interest during the past few years. Such models are of particular interest in suburban areas and macro-cell environments where the scattering is caused by the scatterers around the mobile terminals while the base stations are usually deployed far from scattering [1]. In these environments, the energy transmitted by a single source (mobile terminal) reaches the receiver within a cluster of rays whose distribution depends on the spatial properties of the wireless channel. The nominal AoA and the AS (defined as the standard deviation of the angular deviation around the nominal AoA of a scattered source) of a given source are critical parameters in the design of SDMA systems [1], localization algorithms [2], and optimal detectors [3] in this context.

In [1] and references therein, it has been specified that the AS values encountered in macro-cell environments are typically lower than ten degrees. This fact is desirable since it justifies the recourse to Taylor series expansions to alleviate the complexity of estimating channel parameters. In contrast to the highly complex approaches such as the maximum likelihood [4, 5], and covariance matching [6], a notable simplification has been provided in [7]. Therein, the estimation of the AS and the nominal AoA of a scattered source has been transformed into the localization of two rays symmetrically positioned around the nominal AoA. Subsequently, a classical localization algorithm has been used to estimate both "virtual" AoAs and deduce the required parameters. The focus in [7] has been on root-MUSIC [8] which was shown to achieve better accuracy with relatively low computational complexity compared to some other classical point-source localization algorithms. Nevertheless, it is has been previously found that the performance of such algorithm deteriorates as the angular separation between the sources of interest decreases [9]. This fact becomes more significant when few sensors are deployed. Hence, the utilization of this algorithm to localize both rays in this context is somehow inappropriate in practical situations where the receiving end is equipped with few sensors due to space or cost constraints.

This work is motivated by the need to develop a low-complexity and accurate technique that estimates the channel parameters in practical situations of a locally scattered source with a limited number of sensors. To this end, we take advantage of the approximative form of the observations' covariance matrix proposed in [7] using the Taylor series expansions to retrieve new simple and accurate closed-form estimators of the nominal AoA and the AS. Numerical examples show the efficiency of the proposed approach.

2. PROBLEM STATEMENT AND ASSUMPTIONS

We assume a stationary, ergodic, and narrow-band source $s(t)$ scattered by a large number of local scatterers generating L wavefronts. A ULA composed of M sensors is deployed at the receiver to collect the L replicas of the transmitted signal. The observation vector, $\mathbf{x}(t)$, is then:

$$\begin{aligned} \mathbf{x}(t) &= s(t)\sum_{l=1}^{L}\gamma_l(t)\mathbf{a}[\theta+\tilde{\theta}_l(t)]+\mathbf{b}(t) \\ &\triangleq s(t)\mathbf{h}(t)+\mathbf{b}(t) \end{aligned} \quad (1)$$

where $\mathbf{b}(t) \triangleq [b_1(t) \ldots b_M(t)]^T$ is an unknown noise vector composed of M white Gaussian i.i.d centered stationary signals with variance σ_b^2. γ_l is a random variable representing the channel gain associated with the l^{th} ray. $\tilde{\theta}_l$ is the angular deviation of the l^{th} ray with respect to (w.r.t) the nominal AoA θ. For a ULA of sensors, the entries of $\mathbf{a}$ are:

$$a_m(\theta) = e^{j2(m-1)\pi\kappa\sin(\theta)}; \forall\, 1 \le m \le M. \quad (2)$$

In the sequel, we will omit the time index, t, and use the notation $\omega \triangleq 2\pi\kappa\sin(\theta)$ where κ is the sensors separation in

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wavelengths. The angular deviation is described by a random variable $\tilde{\theta}$ whose discrete realizations are $\tilde{\theta}_l$ in the aforementioned data model. This random variable is assumed to have a symmetrical distribution [7], centered, and with a standard deviation σ_θ. Our aim is to estimate σ_θ and θ. by processing the entries of the following covariance matrix:

$$\mathbf{R_x} \triangleq \mathrm{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{R} + \sigma_b^2\mathbf{I} \quad (3)$$

where $\mathbf{R} \triangleq \sigma_s^2\mathbf{R_h}$, $\mathbf{R_h} \triangleq \mathrm{E}\{\mathbf{h}\mathbf{h}^H\}$, σ_s^2 denotes the source power, and σ_b^2 is the noise power.

3. APPROXIMATIONS

To have an insight on the angular spread effect on source localizers or detectors in the context of macrocell environments, a common trend has been to consider Taylor series expansions. This trend is motivated by the fact that the ASs encountered in these environments have typically small values. Specifically, first-order Taylor series expansion has been used to express the nth spatial frequency as [2, 3, 7]:

$$\begin{aligned} 2\pi\kappa\sin(\theta+\tilde{\theta}) &\approx 2\pi\kappa\sin(\theta) + 2\pi\kappa\tilde{\theta}\cos(\theta) \\ &\triangleq \omega + \tilde{\omega}, \end{aligned} \quad (4)$$

where $\tilde{\omega}$ is the spatial frequency deviation resulting from the angular deviation. According to the previous representation, $\tilde{\omega}$ and $\tilde{\theta}$ have approximately the same probability density function up to a scale factor. One can also establish as in [7] that the standard deviation of $\tilde{\omega}$ corresponding to the nth source is expressed as:

$$\sigma_\omega = 2\pi\kappa\cos(\theta)\sigma_\theta. \quad (5)$$

Hence, determining θ and σ_θ amounts to estimating ω and σ_ω $\forall\, n \in \{1, ..., N\}$. Now, using this first-order Taylor series expansion, it can be established that $\mathbf{R_h}$ is expressed as:

$$\mathbf{R_h} \approx \mathbf{a}(\omega)\mathbf{a}^H(\omega) \odot \mathbf{\Xi}(\sigma_\omega) \quad (6)$$

where $\odot$ denotes the Schur-Hadamard product, and $\mathbf{\Xi}(\sigma_\omega) = \mathbf{R_h}$ when $\omega = 0$. Letting ζ_χ denote the characteristic function of a given random variable χ, the (p, r)th entry of $\mathbf{\Xi}(\sigma_\omega)$ is expressed as:

$$[\mathbf{\Xi}(\sigma_\omega)]_{pr} \approx \zeta_{\tilde{\omega}}\left[(p-r)\sigma_\omega\right], \quad (7)$$

with $\zeta_{\tilde{\omega}}$ being the characteristic function of $\tilde{\omega}$. In [7], a second-order Taylor series expansion of $\mathbf{a}$ and an approximation of order $O\left(\mathrm{E}\{\tilde{\omega}^4\}\right)$ were utilized jointly with the source incoherent distribution to approximate $\mathbf{R_h}$ as:

$$\mathbf{R_h} \approx \frac{1}{2}\mathbf{A}(\omega+\sigma_\omega, \omega-\sigma_\omega)\mathbf{A}^H(\omega+\sigma_\omega, \omega-\sigma_\omega), \quad (8)$$

where

$$\mathbf{A}(\omega+\sigma_\omega, \omega-\sigma_\omega) = \left[\mathbf{a}(\omega+\sigma_\omega) \quad \mathbf{a}(\omega-\sigma_\omega)\right], \quad (9)$$

leading to:

$$\mathbf{R} \approx \frac{\sigma_s^2}{2}\mathbf{A}(\omega+\sigma_\omega, \omega-\sigma_\omega)\mathbf{A}^H(\omega+\sigma_\omega, \omega-\sigma_\omega). \quad (10)$$

The approximation in (8)-(10) is notable. Indeed, the resulting representation is independent of the angular distribution. Rather, it explicitly depends on the nominal AoA and the AS only. More importantly, the originally complicated angular spread estimation problem is transformed into a simpler task consisting in recovering two AoAs. Then, a point source localization algorithm could be used to solve this problem. In [7], it has been stated that the application of this localization algorithm to $\mathbf{\Xi}(\sigma_\omega)$ leads to two symmetrical values $\{\lambda(\sigma_\omega), -\lambda(\sigma_\omega)\}$ where λ is a monotonous positive function which has no analytical expression, but can be determined using a lookup table. However, one can empirically notice that for low σ_ω values, $\lambda(\sigma_\omega) \approx \sigma_\omega$ (cf. Figure 1). For the sake of clarity, this assumption will be made in the sequel.

In [7], the focus has been on root-MUSIC to estimate both AoAs leading to the so-called "spread root-MUSIC." Though it has been stated that this algorithm is better performing than some other classical point-source localization techniques, one should note that the performance of the former deteriorates as the angular separation between a couple of uncorrelated sources of interest (to localize separately) decreases especially in adverse conditions: few sensors, low SNR, and closely-spaced sources. This behavior is due to the fact that the subspace decomposition is no longer easy to perform (the steering matrix is almost rank deficient and/or the noise level is high) [9]. Such situations can be encountered in real-world systems where the aim is to estimate small values of the AS (or equivalently the AoAs of both closely-spaced virtual rays) using a limited number of sensors due to space or cost constraints. In the following, we propose new simple and accurate estimators of both parameters.

4. PARAMETERS ESTIMATION

The proposed closed-form estimators are based on the explicit expression of the sub-diagonal elements of the approximation of $\mathbf{R}$ in (9)-(10) that will be denoted d_m with $m \in \{0, ..., M-1\}$ in the sequel. Before going further, some practical considerations must be pointed out.

4.1. Practical considerations

In practice $\mathbf{R}$ is not available. Rather, we estimate this matrix using a finite number, T, of samples of the observed signals. Let $\mathbf{X} = [\mathbf{x}(1)\ \mathbf{x}(2)\ ...\ \mathbf{x}(T)]$ denote the $M \times T$ matrix representing the available observations data block. Then, the covariance matrix $\mathbf{R_x}$ is approximated as:

$$\hat{\mathbf{R}}_\mathbf{x} = \frac{1}{T}\mathbf{X}\mathbf{X}^H. \quad (11)$$

According to approximation (9)-(10), the signal subspace is two-dimensional. Hence one can estimate the noise power, σ_b^2, by averaging over the $M-2$ smallest eigenvalues of $\hat{\mathbf{R}}_\mathbf{x}$. Using (3), one can estimate $\mathbf{R}$ as:

$$\hat{\mathbf{R}} = \hat{\mathbf{R}}_\mathbf{x} - \hat{\sigma}_b^2\mathbf{I} \quad (12)$$

where $\hat{\sigma}_b^2$ is the estimate of σ_b^2. In addition, $\mathbf{R}$ has an almost Toeplitz structure. Hence, to obtain better estimates of its subdiagonal elements, denoted $d_m, m \in \{0, ..., M-1\}$ in the sequel, one has to average over the sub-diagonal elements of $\hat{\mathbf{R}}$. In other words d_m is estimated as:

$$\hat{d}_m = \frac{1}{M-m} \sum_{k=1}^{M-m} \hat{\mathbf{R}}(k+m, k). \qquad (13)$$

4.2. New Closed-Form Estimators

Using (9)-(10), for $m \in \{0, ..., M-1\}$, d_m is expressed as:

$$d_m = 2\sigma_s^2 e^{jm\omega} \cos(m\sigma_\omega) = d_0 e^{jm\omega} \cos(m\sigma_\omega). \qquad (14)$$

Then, using the following least square fitting:

$$\hat{\omega},\ \hat{\sigma}_\omega = \arg\max_{\omega,\ \sigma_\omega} |\hat{d}_m - d_m|^2 \qquad (15)$$

for $m \in \{1, ..., M-1\}$, we deduce a new expression for $\hat{\omega}$ and $\hat{\sigma}_\omega$ as:

$$\hat{\omega} = \frac{1}{m}\text{angle}(\hat{d}_m) \pm \frac{2p\pi}{m}, \qquad (16)$$

$$\hat{\sigma}_\omega = \frac{1}{m} \arccos\left(\frac{\hat{d}_m}{\hat{d}_0} e^{-jm\hat{\omega}}\right), \qquad (17)$$

where $p \in \{0, ..., \lfloor \frac{m}{2} \rfloor\}$ and $\lfloor . \rfloor$ is the integer part operator.

*Remark 1*_ One should note that $\hat{\omega}$ can be determined without resorting to the approximation (8)-(9), but up to a potential $\pi-$phase indetermination. Indeed, the entries of the matrix $\Xi(\sigma_\omega)$ (which are the Fourier transforms of the distribution of the angular deviation) defined in (6)-(7) are real valued since the angular deviation has a symmetrical distribution. Nevertheless, thanks to the approximation (8)-(9), we can confirm that the $\pi-$phase indetermination does not appear for low σ_ω values [cf. the following condition in (18)].

*Remark 2*_ Estimator (17) requires a prior knowledge of the range of σ_ω. Indeed, using (17), we suppose that:

$$\sigma_\omega \leq \frac{\pi}{2m}. \qquad (18)$$

Note that this condition is satisfied for real-world applications where the AS keeps relatively low values (e.g., in macrocell environments) and the number of sensors is limited due to cost or space constraints. Furthermore, one can always start with the lowest values of m where the condition (18) is not that restrictive.

5. SIMULATION RESULTS

Along our simulations, we will use the root mean squared error (RMSE) $\text{RMSE}(\varphi) = \sqrt{\frac{1}{MC}\sum_{m=1}^{MC} \left|\varphi - \hat{\varphi}^{(m)}\right|^2}$, as a performance index, where φ is the parameter to estimate, and $\hat{\varphi}^{(m)}$ is its estimate at the mth Monte-Carlo run ($1 \leq m \leq MC$). In all of the investigated scenarios, we take $MC = 5.10^2$. We compare the proposed method with "spread root-MUSIC", where root-MUSIC is employed to determine the two symmetrical AoAs. We implement the data model (1) with a ULA of 3 sensors, a BPSK source located at $\theta = 0°$, a Gaussian distributed angular deviation with σ_θ as a standard deviation, $L = 50$ randomly generated replicas of the source signal, and complex Gaussian centered i.i.d random variables $(\gamma_l)_{1 \leq l \leq L}$ such that $\text{E}\{|\gamma_l|^2\} = 1/L$.

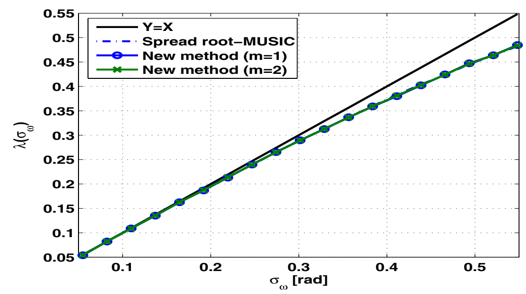

Fig. 1. $\lambda(\sigma_\omega)$ vs. σ_ω.

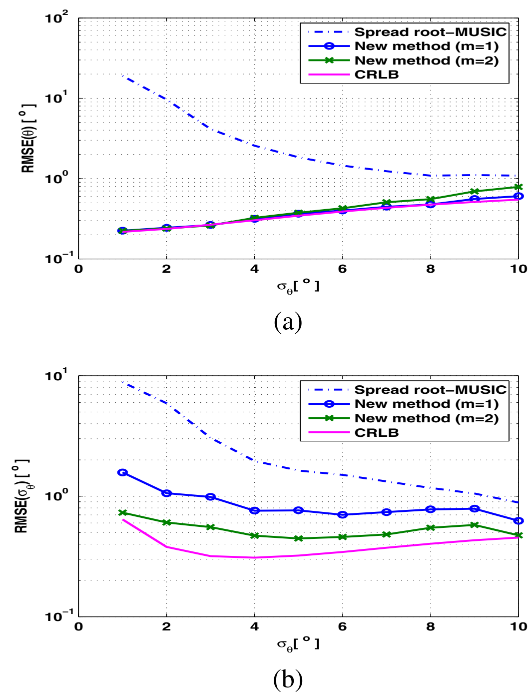

Fig. 2. RMSE vs. σ_θ at SNR= 10 dB: (a) RMSE(θ), (b) RMSE(σ_θ).

In Fig. 2, we plot the variations of the RMSE w.r.t σ_θ for an SNR = 10 dB and $T = 2.10^2$. Notice first how the performance of spread root-MUSIC deteriorates for low AS values. This fact is due to the inability of root-MUSIC to separate two closely-spaced point sources with a limited number of sensors. In contrast, our new method achieves good accuracy (using the first, $m = 1$, or the second, $m = 2$, subdiagonal). In Fig. 3, we check the effect of the SNR on the RMSE for $\sigma_\theta = 3°$ and $T = 2.10^2$. We see again the same behavior of performance collapse for spread root-MUSIC at low SNR values. On the other hand, the proposed closed-form expressions keep a regular behavior and by far outperform spread root-

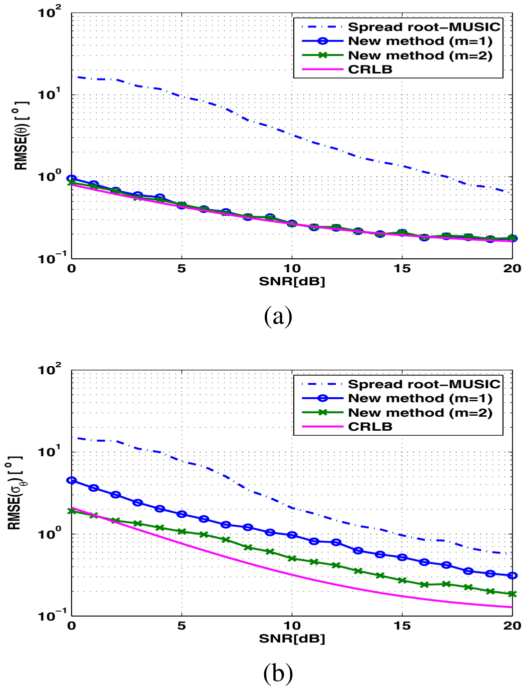

Fig. 3. RMSE vs. SNR at $\sigma_\theta = 3°$: (a) RMSE(θ), (b) RMSE(σ_θ).

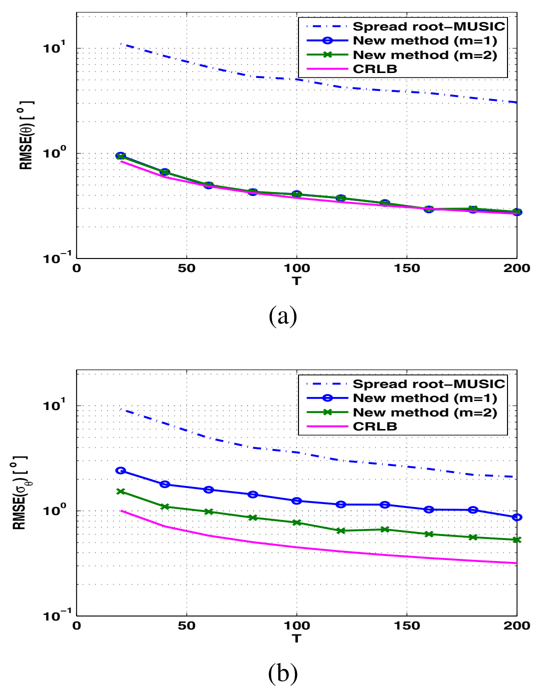

Fig. 4. RMSE vs. T at SNR $= 10$ dB, $\sigma_\theta = 3°$: (a) RMSE(θ) [°], (b) RMSE(σ_θ) [°].

MUSIC especially at relatively low SNR values. In Fig. 4, we plot the variations of the RMSE w.r.t the number of snapshots, T, for $\sigma_\theta = 3°$ and SNR $= 10$ dB. We clearly see that the new technique outperforms spread root-MUSIC. As an example, the latter requires only $T = 20$ snapshots to outperform spread root-MUSIC with $T = 2.10^2$ snapshots. This fact accounts, once again, for the low-computational complexity required by the proposed approach to perform accurately. Finally, it must be stated that the selection of the highest value of m provides more accurate estimates of the AS. To estimate the nominal AoA, both subdiagonal orders seem to achieve the same accuracy for low AS values. As the AS increases, the first subdiagonal seems to provide the best accuracy (cf. Fig. 2).

6. CONCLUSION

We proposed new simple and accurate closed-form estimators of the AS and the nominal AoA of a locally scattered source. Using the Taylor series expansions (for the spatial frequency and the steering vector, respectively), we transformed the estimation of these parameters into the localization of two point sources as it has been suggested in [7]. Next, we used a least square fitting of the entries of the approximate expression of the covariance matrix to derive new estimators for both parameters. The proposed approach is computationally simple and achieves high accuracy even with few sensors and snapshots as demonstrated by simulations.

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