A NARROWBAND APPROACH TO BLIND SOURCE SEPARATION IN CONVOLUTIVE MIMO MIXTURES*

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ABSTRACT

In this contribution, we adopt a novel and simple narrowband approach to blind separation of mutually independent and temporally i.i.d. sources in convolutive MIMO mixtures. After remodelling the observation at the sensor array as an instantaneous mixture where the delayed replicas of the desired signals from multipath propagation are seen as separate sources independent of each other, we estimate them all in an analysis step by a conventional narrowband blind source separation (BSS) technique. In a synthesis step, we match the delayed replicas of each source based on cross-correlation then combine them after proper time & phase alignments and weighting. In the process, this multipath matching and combining procedure identifies the convolutive MIMO channel and is able to provide accurate expressions for direct blind deconvolution (BD) equalizers. Simulations support the efficiency of the new narrowband approach to BD.

1. INTRODUCTION

BSS has gained increasing researchers' interest over the last few years. This fact is due to the inherent goal of the BSS which consists in retrieving unknown sources by processing their mixtures only. Commonly, instantaneous mixtures are considered [1, 2]. However, this assumption is inconceivable in several practical applications. In wideband wireless communications for instance, the transmitted signals reach the receiver with several delays generated by the scatterers around the communication terminals [3]. In such contexts, the channel is generally modelled as a finite impulse response (FIR) filter and the original sources are recovered through BD.

The case of spatially independent sources with temporally i.i.d. sequences has been recently investigated in several works [4, 5, 6, 7]. In [6], Inouye proposed new BD criteria extending the single convolutive channel case investigated by Shalvi and Weinstein in [8]. In [7], Amari et al. proposed a set of online algorithms based on the so-called natural gradient. However, online algorithms generally exhibit slow convergence rate. Fortunately, some batch algorithms have been shown to be able to achieve high accuracy even with a limited number of snapshots [5]. Another interesting work has been recently published in [9]. Therein, Diamantaras at al. considered a less restrictive case of temporally white sources but with at least as many sensors as all signal replicas. The authors propose a new procedure based on a "filter deflation". By performing successive subspace projections, the general BD problem is transformed into an instantaneous BSS problem. Unfortunately, these projections "kill" the multipath components instead of taking advantage of the information contained therein to better estimate the sources and improve the quality of service in radiocommunication applications for instance.

In this paper, we propose a two-stage technique to perform the BD of spatially independent sources with temporally i.i.d. sequences. First, we take advantage of the spatial independence and the temporal i.i.d property of the sources to blindly estimate all the signal replicas using an instantaneous BSS algorithm. Second, we match the resulting replicas and combine them after proper time & phase alignments and weighting to improve source reconstruction. Simulations validate the efficiency of the new technique.

2. DATA MODEL AND ASSUMPTIONS

We consider M temporally i.i.d. and mutually independent sources s_m for $m = 1, \ldots, M$, received by an array of N sensors after propagation through M multipath vector channels quasi-static over T-sample durations and characterized each by L_m paths for $m = 1, \ldots, M$, and a maximum delay spread $\tau_{\max}$. The N-dimensional wideband observation vector resulting from this MIMO convolutive mixture can be written as follows at discrete-time sample index t for $t = 1, \ldots, T$:

$$\begin{aligned} \mathbf{x}(t) &= \sum_{m=1}^{M} \mathbf{b}_m(t) \otimes s_m(t) + \mathbf{n}(t) \qquad (1) \\ &= \sum_{m=1}^{M} \sum_{l=0}^{\tau_{\max}-1} \mathbf{b}_m(l)\, s_m(k-l) + \mathbf{n}(t)\,, \end{aligned}$$

where $\otimes$ denotes convolution, $\mathbf{b}_m(t)$ is the N-dimensional channel vector of the mth source and $\mathbf{n}(t)$ is an N-dimensional additive white Gaussian noise vector independent of all sources s_m for $m = 1, \ldots, M$.

Accounting for the fact that each channel response vector $\mathbf{b}_m(t)$, for $m = 1, \ldots, M$, has L_m multipath components with path delays $\tau_{m,1}, \ldots, \tau_{m,p}, \ldots, \tau_{m,L_m}$ - i.e., we have $\mathbf{b}_m(l = \tau_{m,p}) = \mathbf{b}_{m,p}$ (i.e., for $l \in$

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$\{\tau_{m,1},\ldots,\tau_{m,p},\ldots,\tau_{m,L_m}\}$) and 0 otherwise when l ranges from 0 to $\tau_{\max}-1$ - we can rewrite the observation vector as follows:

$$\mathbf{x}(t)=\sum_{m=1}^{M}\sum_{p=1}^{L_m}\mathbf{b}_{m,p}\, s_m(k-\tau_{m,p})+\mathbf{n}(t)\ , \qquad (2)$$

where $\tau_{m,p}$ denotes the pth propagation path delay of the mth source to the array of sensors. Without loss of generality, we assume $\tau_{m,1}=0$ for $m=1,\ldots,M$.

We define the total number of multipath components over all sources as:

$$K=\sum_{m=1}^{M}L_m\ . \qquad (3)$$

As it will become obvious from the following developments in Sec. 3.2, we require, similarly to [9], the following condition:

$$K\leq N\ ; \qquad (4)$$

meaning that the total number of multipath components over all sources must not exceed the observation's dimension. In practical situations where the number of sensors is limited, the observation's dimension N can be increased beyond the antenna-array's size by oversampling [9]. Without loss of generality, we assume here for simplicity the case where N is equal to the number of sensors (i.e., no oversampling).

3. PROPOSED BLIND DECONVOLUTION TECHNIQUE

Based on narrowband remodelling of the data observation model, we estimate in an analysis step all multipath signal components regardless of the sources they replicate using conventional instantaneous BSS techniques. In a synthesis step, we match all the multipath replicas of each source and combine them after proper time & phase alignments and weighting.

3.1. Narrowband Remodelling

Let us define the two single- and double-index sets with same cardinality K as $\mathcal{I}_1=\{1,\ldots,K\}$ and $\mathcal{I}_2=\{(1,1),\ldots,(1,L_1),\ldots,(M,1),\ldots,(M,L_M)\}$, respectively. Accordingly, with one-to-one notation mapping from one set to the other (i.e., $k\in\mathcal{I}_1\leftrightarrow(m,p)\in\mathcal{I}_2$), we define $\underline{\mathbf{b}}_k=\mathbf{b}_{m,p}$ and $\underline{s}_k(t)=s_m(t-\tau_{m,p})$. We hence rewrite the observation model in (2) as follows:

$$\mathbf{x}(t)=\sum_{k=1}^{K}\underline{\mathbf{b}}_k\, \underline{s}_k(t)+\mathbf{n}(t)=\underline{\mathbf{B}}\ \underline{\mathbf{s}}(t)+\mathbf{n}(t)\ , \qquad (5)$$

where $\underline{\mathbf{B}}=[\underline{\mathbf{b}}_1,\ldots,\underline{\mathbf{b}}_k,\ldots,\underline{\mathbf{b}}_K]$ is a column-wise full rank $N\times K$ propagation channel matrix over all multipath components and sources and $\underline{\mathbf{s}}(t)=[\underline{s}_1(t),\ldots,\underline{s}_k(t),\ldots,\underline{s}_K(t)]^T$ is a $K\times 1$-dimensional vector of desired signal replicas over all multipath components and sources. Since the M desired sources s_m for $m=1,\ldots,M$ are mutually independent and each i.i.d. in time, the K desired signal replicas $\underline{s}_k$ for $k=1,\ldots,K$ are mutually independent variables (i.e., when taken at the sample index t).

1. Initialization
- **1.1** define the total set of indices $\hat{\mathcal{I}}_1=\{1,\ldots,\hat{K}\}$
- **1.2** set $m=0$
- **1.3** define the set of remaining indices $\overline{\mathcal{I}}_1(0)=\hat{\mathcal{I}}_1$ after assignment of multipath indices to previous source

2. Multipath Matching
- **2.1** do the following while $\overline{\mathcal{I}}_1(m)\neq\emptyset$:
  - **2.1.1** set $m=m+1$
  - **2.1.2** for the mth source define:
    - $i_{m,1}$ as first index available in $\overline{\mathcal{I}}_1(m-1)$
    - the set of multipath indexes $\hat{\mathcal{I}}_1(m)=\{i_{m,1}\}$
    - the set of multipath delays $\hat{\mathcal{D}}(m)=\{0\}$
    - the set of multipath ambiguities $\hat{\mathcal{A}}(m)=\{1\}$
  - **2.1.3** for $k\in\overline{\mathcal{I}}_1(m-1)\backslash\{i_{m,1}\}$
    - for $\tau\in\mathcal{T}=\{-\tau_{\max},\ldots,\tau_{\max}\}$ calculate the cross-correlation $\rho_{i_{m,1},k}(\tau)$ between $\hat{\underline{s}}_{i_{m,1}}(t)$ and $\hat{\underline{s}}_k(t-\tau)$
    - define $\overline{\rho}_{i_{m,1},k}=\rho_{i_{m,1},k}(\overline{\tau})$ such that $\overline{\tau}=\arg\max_{\tau\in\mathcal{T}}|\rho_{i_{m,1},k}(\tau)|^2$
    - do the following if $|\overline{\rho}_{i_{m,1},k}|^2>\epsilon_{\min}^2$
      - $\triangleright$ $\hat{\mathcal{I}}_1(m)=\hat{\mathcal{I}}_1(m)\cup\{k\}$
      - $\triangleright$ $\hat{\mathcal{D}}(m)=\hat{\mathcal{D}}(m)\cup\{\overline{\tau}\}$
      - $\triangleright$ $\hat{\mathcal{A}}(m)=\hat{\mathcal{A}}(m)\cup\left\{a=\frac{\overline{\rho}i_{m,1},k}{|\overline{\rho}_{i_{m,1},k}|}\right\}$
  - **2.1.4** estimate the number of multipaths for mth source as $\hat{L}_m=\text{Card}[\hat{\mathcal{I}}_1(m)]$
  - **2.1.5** adopt accordingly the following notation:
    - $\hat{\mathcal{I}}_1(m)=\{i_{m,1},\ldots,i_{m,\hat{L}_m}\}$
    - $\hat{\mathcal{D}}(m)=\{\hat{\tau}_{m,1},\ldots,\hat{\tau}_{m,\hat{L}_m}\}$
    - $\hat{\mathcal{A}}(m)=\{\hat{a}_{m,1},\ldots,\hat{a}_{m,\hat{L}_m}\}$
  - **2.1.6** define $\overline{\mathcal{I}}_1(m)=\overline{\mathcal{I}}_1(m-1)\backslash\hat{\mathcal{I}}_1(m)$
- **2.2** estimate the number of sources as $\hat{M}=m$

3. Multipath Combining

for $m=1,\ldots,\hat{M}$

$$\hat{s}_m(t)=\sum_{p=1}^{\hat{L}_m}\hat{a}_{m,p}\ \hat{\lambda}_{i_{m,p}}^2\ \hat{\underline{s}}_{i_{m,p}}(t+\hat{\tau}_{m,p}-\tau_0)$$

Fig. 1. Main algorithmic steps of the proposed mulipath matching and combining procedure.

The data observation model reformulated in (5) is now typically of a narrowband MIMO mixture[1]. Without loss of generality, we assume that all sources $\underline{s}_k$ for $k=1,\ldots,K$ have the same unit power and transfer all normalization factors into the norms of $\underline{\mathbf{b}}_k$, respectively.

3.2. Analysis: Conventional Narrowband BSS

As outlined previously, many BSS algorithms are available in the literature to handle the problem of extracting the source vector $\underline{\mathbf{s}}(t)$ out of an observation stemming from instantaneous MIMO mixtures such as $\mathbf{x}(t)$ of (5). Here we exploit the algorithm in [1, 2] for its great efficiency and suitability.

[1] Immediate application of numerous narrowband techniques such as high-resolution source localization methods over convolutive mixtures become *ad hoc* with the reformulated data model in (5).

After estimation of the signal subspace rank which coincides with the estimated number of sources $\hat{K}$ if $K \leq N$ [hence the condition in (4)] and projection of $\mathbf{x}(t)$ over the signal subspace, the algorithm in [1] provides an estimate of $\hat{\underline{\mathbf{B}}}$ within a permutation matrix and complex ambiguities over columns that have little impact on the proposed approach as it will become clear in the following subsection. From $\hat{\underline{\mathbf{B}}}$, we form the zero-forcing (ZF) detector (alternative detectors such as MMSE could be applied instead) given by:

$$\underline{\mathbf{W}} = \hat{\underline{\mathbf{B}}} \left(\hat{\underline{\mathbf{B}}}^H \hat{\underline{\mathbf{B}}} \right)^{-1} , \tag{6}$$

and hence estimate the source vector as follows:

$$\hat{\underline{\mathbf{s}}}(t) = \underline{\mathbf{W}}^H \mathbf{x}(t) \ . \tag{7}$$

Before we explain the proposed multipath matching and combining procedure in the following subsection, we define the multipath power used there as follows for $k = 1, \ldots, \hat{K}$:

$$\hat{\lambda}_k^2 = \|\hat{\underline{\mathbf{b}}}_k\|^2 \ . \tag{8}$$

3.3. Synthesis: Multipath Matching and Combining

The proposed multipath matching and combining procedure is summarized in Fig. 1. In the matching step, we cluster together the indices $k \in \hat{\mathcal{I}}_1 = \{1, \ldots, \hat{K}\}$ (see step 1.1) of the multipath signal components replicating the same source based on cross-correlation.

We proceed sequentially with one source at a time. Let us assume that we have already assigned indices up to the $(m-1)$th source and that the set of indices specifically allocated to the $(m-1)$th source is referred to as $\hat{\mathcal{I}}_1(m-1)$. Then $\overline{\mathcal{I}}_1(m) = \hat{\mathcal{I}}_1 \backslash \cup_{q=1}^{m-1} \hat{\mathcal{I}}_1(m-1)$ (see steps 1.3 and 2.1.6) designates the remaining indices (i.e., not assigned yet in $\hat{\mathcal{I}}_1$). As long as this set is not empty (see step 2.1), there are more sources to extract and multipath components to which they must be matched. In this case, we move on to matching the multipath components of the mth source (see steps 1.2 and 2.1.1).

Let us assign $s_{i_{m,1}}$ where $i_{m,1}$ designates the first index available in $\overline{\mathcal{I}}_1(m)$ as the first delayed replica of the mth source. As a reference signal, it has an ambiguity of 1 and a delay of 0 when cross-correlated with itself (see step 2.1.2). However, with the other remaining components not assigned yet (see step 2.1.3), it has a cross-correlation function $\rho_{i_{m,1},k}(\tau)$ for $\tau \in \mathcal{T} = \{-\tau_{\max}, \ldots, \tau_{\max}\}$ where $\tau_{\max}$ is the maximum delay spread in sample durations. Let us define $\overline{\tau}$ as the value of the time delay $\tau \in \mathcal{T}$ that maximizes the cross-correlation power and $\overline{\rho}_{i_{m,1},k} = \rho_{i_{m,1},k}(\overline{\tau})$ as the value of the cross-correlation function at that time delay (see step 2.1.3). If $|\overline{\rho}_{i_{m,1},k}|^2$ exceeds a minimum cross-correlation power threshold for multipath matching, then the kth signal component is assigned as a delayed replica of the mth source. Hence the set of indices, delays and ambiguities of the mth source $\hat{\mathcal{I}}_1(m)$, $\hat{\mathcal{D}}(m)$ and $\hat{\mathcal{A}}(m)$ are augmented (see step 2.1.3), respectively, with $\{i_{m,1}\}$, $\{\overline{\tau}\}$ and $a = \overline{\rho}_{i_{m,1},k}/|\overline{\rho}_{i_{m,1},k}|$. Once all remaining indices in $\overline{\mathcal{I}}_1(m)$ are processed, an estimate of the number of multipaths for the mth source can be provided by the equal cardinality of either set, e.g., $\hat{L}_m = \text{Card}[\hat{\mathcal{I}}_1(m)]$. This allows moving on to matching the components of the next source, etc. Once all indices in $\overline{\mathcal{I}}_1(m)$ are assigned [i.e., $\overline{\mathcal{I}}_1(m) = \emptyset$] thereby terminating the multipath matching step, the last incremented value of m provides an estimate of the number of sources $\hat{M}$ (see step 2.2).

In a final synthesis step (see step 3), the source vector estimate $\hat{\underline{\mathbf{s}}}_k$ in (7) and the multipath power estimates $\hat{\lambda}_k^2$ in (8) for $k = 1, \ldots, \hat{K}$, along with the sets of indices, delays and ambiguities $\hat{\mathcal{I}}_1(m)$, $\hat{\mathcal{D}}(m)$, and $\hat{\mathcal{A}}(m)$ for $m = 1, \ldots, \hat{M}$ allow for efficient reconstruction of the original sources within complex ambiguities by efficiently combining their aligned replicas. For simplicity, we recur in step 3 to maximum ratio combining (MRC) and introduce an arbitrary delay $\tau_0 > \tau_{\max}$ for causal processing.

Coming back to the original convolutive data model of (1), we can now reconstruct the equivalent equalizers which directly implement BD as follows for $m = 1, \ldots, \hat{M}$:

$$\mathbf{w}_m(t) = \sum_{p=1}^{\hat{L}_m} \hat{a}_{m,p} \, \hat{\lambda}_{i_{m,p}}^2 \underline{\mathbf{w}}_{i_{m,p}} \, \delta(t + \hat{\tau}_{m,p} - \tau_0) \ , \tag{9}$$

where $\underline{\mathbf{w}}_k$ is the kth column of $\underline{\mathbf{W}}$ in (6). We hence rewrite step 3 as follows for $m = 1, \ldots, \hat{M}$:

$$\begin{aligned} \hat{s}_m(t) &= \mathbf{w}_m^H(t) \otimes \mathbf{x}(t) \\ &= \sum_{m'=1}^{M} \left[\mathbf{w}_m^H(t) \otimes \mathbf{b}_{m'}(t)\right] \otimes s_{m'}(t) + \mathbf{w}_m^H(t) \otimes \mathbf{n}(t) \\ &= \sum_{m'=1}^{M} z_{m,m'}(t) \otimes s_{m'}(t) + n'_m(t) \ , \end{aligned} \tag{10}$$

where $z_{m,m'}(t)$ denotes the time response of the mth equalizer to the propagation channel of the m'th source and $n'_m(t)$ is the output noise of the mth equalizer. Plots of $z_{m,m'}(t)$ will illustrate the performance of BD in the following simulations section.

4. SIMULATION RESULTS

We consider two i.i.d. and mutually independent BPSK sources (i.e., $M = 2$) propagating each through three-path channels (i.e., $L_m = L = 3$) to an array of six sensors (i.e., $N = K = M \times N = 6$). Without loss of generality, the delays for both sources are fixed to 0, 1, and 2. We hence set the arbitrary delay for causal processing to $\tau_0 = 3$ (see Sec. 3.3). Statistics are calculated over blocks of $T = 10^3$ snapshots [1] (see Sec. 3.2). We set the power threshold to $\epsilon_{\min}^2 = 0.2$ (see Sec. 3.3). The randomly-generated real-valued[2] channel taps are given below:

$$[\mathbf{b}_{1,1} \ \mathbf{b}_{1,2} \ \mathbf{b}_{1,3}] = \begin{pmatrix} -0.5533 & -0.0263 & -0.5241 \\ -0.1943 & +0.8188 & -0.0229 \\ -0.0730 & +1.5648 & +1.1664 \\ -0.5344 & +0.4845 & -0.2880 \\ -0.0615 & +0.0868 & -0.5717 \\ -0.8864 & -1.1519 & +0.3011 \end{pmatrix},$$

[2]For illustration purposes, we considered real-valued signals only.

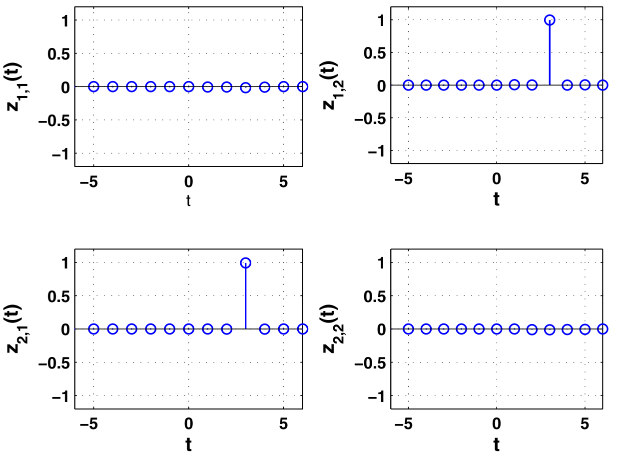

Fig. 2. Time response $z_{m,m'}(t)$ of the mth equalizer to the propagation channel of the m'th source [see (10)].

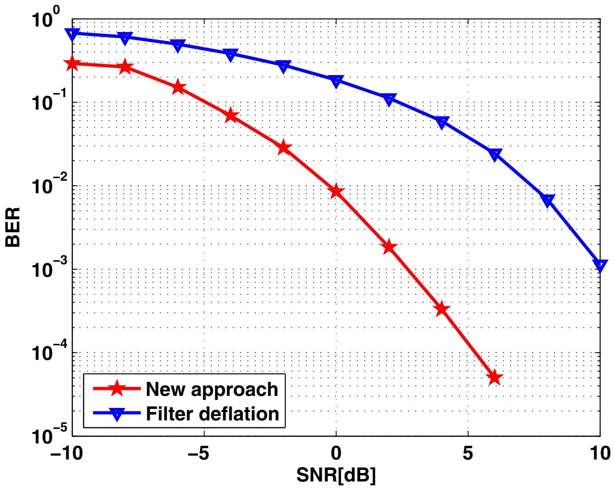

Fig. 3. BER vs. SNR.

$$[\mathbf{b}_{2,1}\ \mathbf{b}_{2,2}\ \mathbf{b}_{2,3}] = \begin{pmatrix} -0.1539 & +0.7615 & -0.0048 \\ +0.8548 & +2.5766 & -1.9293 \\ +1.0872 & +1.3200 & +0.7305 \\ -1.0818 & -0.4889 & -0.9597 \\ -0.1148 & -0.3357 & +0.2281 \\ +1.3835 & -0.3027 & -1.5651 \end{pmatrix}.$$

In Fig. 2, we plot $z_{m,m'}(t)$ in (10) at an SNR of 20 dB to assess the ability of the proposed wideband BSS algorithm to separate the sources. Obviously, the proposed algorithm is able to implement BD very accurately within a source permutation. The response of the two equalizers to one source is a perfect time delay at $t = \tau_0$ and a perfect null to the other and vice-versa, respectively.

To further illustrate the efficiency of the proposed algorithm, we compare its performance with the recent "filter deflation" algorithm proposed in [9] where all the taps but one are "killed" after successive cancelling subspace projections. To the best of our knowledge, this is the only technique that came closest to exploring a narrowband approach to BD, under the same condition in (4) with however a very different concept. Hence its choice as a benchmark. As a performance index, we measure the bit error rate (BER) over 10^2 Monte-Carlo runs for the T snapshots. In contrast to the proposed approach, we assume for the "filter deflation" algorithm that the length of the channel response is known.

In Fig. 3, we plot the mean BER over both sources vs. SNR achieved by both techniques over the same channels given above. The proposed approach clearly achieves very significant SNR gains over the "filter deflation" technique. This result can be expected since the proposed method takes advantage of the diversity offered by the multipath phenomenon after matching all the time-shifted versions of the original signals and combining them to enhance the signals' reconstruction. In contrast, the filter deflation approach kills all the multipath components but one for every source.

5. CONCLUSION

In this paper, we proposed a new and simple narrowband approach to BD of mutually independent and temporally i.i.d. sources. Based on narrowband remodelling of the data observation model, we estimate in an analysis step all multipath signal components regardless of the sources they replicate using conventional BSS techniques. In a synthesis step, we match all the multipath replicas of each source and combine them after proper time & phase alignments and weighting. Simulations validate the efficiency of this novel approach.

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