Iterative SNR Estimation for MPSK Modulation over AWGN Channels

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***Abstract*— Non-data-aided (NDA) SNR estimation is considered for M-PSK transmission over additive white Gaussian noise channels. An iterative SNR estimator is derived from an envelope-based estimator. The performance of the proposed estimator is compared to existing algorithms. Simulation results shown for QPSK and 8-PSK demonstrate that the new estimator clearly performs as well as the best SNR estimator in each SNR range.**

I. INTRODUCTION

Various digital communication applications, such as power control, bit error estimation, and turbo decoding, involve the knowledge of the Signal-to-noise ratio (SNR). For optimal performance, SNR estimation must be as accurate as possible. Several SNR estimation techniques have been proposed for AWGN channels. These estimators can be divided into two classes. One class is for data-aided estimators which assume the knowledge of the transmitted data, or that the transmitted data can be reconstructed from the received data and used by the estimator as if it was perfectly reconstructed. The other class is for non-data-aided estimators. For this class of estimators, the transmitted data remain unknown to the receiver. A comparison of Data Aided (DA) and Non-data-aided (NDA) estimators was performed in [1]. Although the pilot data-aided estimators perform better, the main advantage of the NDA estimators is their bandwidth efficiency due to the elimination of training sets. In [1], as in many other studies, only the case of coherent channels is studied and thus no phase ambiguity is considered. In the case of incoherent channels, contrary to other estimators, the NDA envelope-based estimators are still operable without phase recovery. Hence, in this paper, we consider only non-data-aided envelope-based SNR estimators. For this class of estimators, several SNR estimation algorithms have been investigated. In [4], the authors introduce the Amplitude Moment (AM) estimator for PSK modulation over AWGN channels. The AM estimator is based on the first and the second absolute moments. Although this estimator emerges as the best candidate at low SNR, it suffers from a degradation in performances at high SNR values where it is outperformed by the $\mathrm{M_2M_4}$ estimator [2], which is based on the second and fourth moments. In [6], the authors present a class of estimators for QAM constellations. For PSK constellations, the method proposed in [6] can outperform the $\mathrm{M_2M_4}$ method with appropriate parameter selection.

In this paper, an iterative estimation method is presented for M-PSK modulation over AWGN complex channels. The new estimator outperforms other estimators in terms of NMSE (normalized mean squared error) and, since it is an envelope-based estimator, phase recovery is not needed.

Simulation results demonstrate that the new estimator exhibits good performance over wider SNR ranges than other existing algorithms. Simulation results shown for QPSK and 8-PSK, demonstrate that the new estimator clearly performs as well as the best SNR estimator in each SNR range.

The paper is organized as follows. Section II introduces the system model. The various SNR estimators under consideration are described in Section III. In Section IV, we derive a new envelope-based NDA estimator. In Section V, Monte Carlo simulation results are provided for the new estimator and compared with other methods. Section VI concludes the paper.

II. SYSTEM MODEL

We consider MPSK modulation over frequency-flat fading channels. Symbol-spaced samples at the matched filter output are given by

$$y_n = Ae^{j\phi}a_n + w_n, \tag{1}$$

where $n = 1, 2, ...N$ is the time index in the observation interval, y_n is the received signal, $A\exp(j\phi)$ is the channel coefficient assumed complex and constant over the observation interval, a_n is the transmitted MPSK signal and w_n is a realization of a zero mean complex white Gaussian random process of variance $N_0 = 2\sigma^2$. The SNR of the received symbol is given by:

$$\rho = \frac{A^2}{2\sigma^2} = \frac{S}{N_0}. \tag{2}$$

As mentioned above, we are currently interested in envelope-based estimators, which do not assume the knowledge of the channel phase. From our system model in (1), it can be shown that the probability density function (PDF) for $|y_n|$ is expressed as follows:

$$f_{|y_i|}(y) = \frac{y}{\sigma^2}\exp\left(-\rho - \frac{y^2}{2\sigma^2}\right)\mathrm{I}_0\left(y\sqrt{\frac{2\rho}{\sigma^2}}\right), \tag{3}$$

where $\mathrm{I}_0()$ is the Bessel function of the first kind and of order 0. The first absolute moment of the Ricean variable $|y_n|$ is given by [5]:

$$M_1 = \mathrm{E}(|y_n|) = (2\sigma)^{1/2}\Gamma(3/2)\exp(-\rho)\mathrm{M}(3/2;1;\rho), \quad (4)$$

where $\mathrm{M}(.;.;.)$ is the confluent hypergeometric function and Γ is the gamma function.

III. Existing SNR Estimation Methods

A. The amplitude Moments Method

In [4], the amplitude moment (AM) estimator, an SNR estimator using the first and second moments, is proposed. Using eq. (4) and

$$M_2 = \mathrm{E}\{|y_n|^2\} = S + N_0, \quad (5)$$

leads to the following expression:

$$\frac{M_1^2}{M_2} = \frac{\pi}{4}\left(1 - \frac{S}{M_2}\right)\mathrm{M}\left(-1/2;1;-\frac{S/M_2}{1-S/M_2}\right)^2, \quad (6)$$

from which S, the desired signal power, can be computed. N_0 is simply:

$$N_0 = M_2 - S. \quad (7)$$

In practice, the AM estimate $\hat{\rho}_{AM}$ is obtained by substituting M_1 and M_2 in eq. (6) and (7) by sample average versions:

$$\hat{M}_1 = \frac{1}{N}\sum_{n=1}^{n=N}|y_n|, \quad (8)$$

and

$$\hat{M}_2 = \frac{1}{N}\sum_{n=1}^{n=N}|y_n|^2, \quad (9)$$

For a determinate value of $\frac{M_1^2}{M_2}$, using a lookup table, we obtain an approximate estimate for S by interpolation (See Fig. 1).

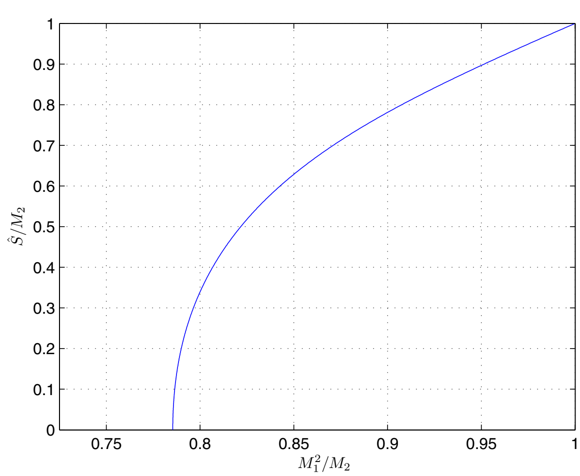

Fig. 1. $\hat{S}/M_2$ vs.M_1^2/M_2.

The major disadvantage of this method is that, due to the finite dimension of the lookup table, the bias tends to increase with the SNR. This effect is more pronounced with smaller lookup table sizes.

B. The M_2M_4 Method

In order to estimate the SNR, the $\mathrm{M}_2\mathrm{M}_4$ method [2] uses eq. (5) and the fourth moment of $|y_n|$:

$$M_4 = k_a S^2 + 4SN_0 + k_w N_0^2, \quad (10)$$

where $k_a = \mathrm{E}\{|a_n|^4\}/\mathrm{E}\{|a_n|^2\}^2$ and $k_w = \mathrm{E}\{|w_n|^4\}/\mathrm{E}\{|w_n|^2\}^2$ are the kurtosis of the complex signal and the complex noise. For MPSK signal over complex AWGN channels we have $k_a = 1$ and $k_w = 2$. Hence, (10) simplifies to

$$M_4 = S^2 + 4SN_0 + 2N_0^2. \quad (11)$$

Combining eq. (5) and eq. (11), the $\mathrm{M}_2\mathrm{M}_4$ estimator reduces to:

$$\rho_{M_2M_4} = \frac{\sqrt{2M_2^2 - M_4}}{M_2 - \sqrt{2M_2^2 - M_4}}. \quad (12)$$

The estimate $\hat{\rho}_{M_2M_4}$ is obtained by substituting M_4 and M_2 in eq. (12) by the sample average defined in eq. (9) and:

$$\hat{M}_4 = \frac{1}{N}\sum_{n=1}^{n=N}|y_n|^4. \quad (13)$$

$\mathrm{M}_2\mathrm{M}_4$ outperforms AM at high SNR, nevertheless AM is superior at low SNR.

C. The NDA envelope-based SNR estimation approach presented in [6]

For the sake of simplicity, we choose to refer to the NDA envelope-based SNR estimation approach presented in [6] by denoting it simply by the name "Gao method". We present in this section the Gao method applied to M-PSK signals.

Let us define the kth moment of $|y_n|$ as:

$$M_k = \mathrm{E}\left\{|y_n|^k\right\}. \quad (14)$$

For PSK modulation, M_k is the kth moment of a Ricean variable:

$$M_k = (2\sigma)^{k/2}\,\Gamma\left(k/2+1\right)\exp\left(-\rho\right)\mathrm{M}\left(k/2+1;1;\rho\right), \quad (15)$$

We see from (15), that the moments depend on two unknown parameters: the SNR, ρ, and the noise variance $2\sigma^2$. Hence, a moment-based SNR estimator requires estimates of at least two different moments. Suppose that, for $k \neq l$, we define the following functions of ρ:

$$f_{k,l}\left(\rho\right) \triangleq \frac{M_k^l(\sigma^2,\rho)}{M_l^k(\sigma^2,\rho)}, \quad (16)$$

which depend on ρ but not on σ. Then we can construct moment-based estimators for ρ, which are expressed as

$$\hat{\rho}_{k,l} \triangleq f_{k,l}^{-1}\left(\frac{\widehat{M_k^l}}{\widehat{M_l^k}}\right), \quad (17)$$

where $\widehat{M_k} \triangleq \frac{1}{N}\sum_{n=1}^{N}|y(n)|^k$.

Although the analytical inversion of $f_{k,l}(\cdot)$ is often not tractable, we can easily implement this estimator by a look-up table. However, for $k=2$ and $l=4$, the estimator yields a closed-form solution which is actually equivalent to the $\mathrm{M_2M_4}$. In [6], simulation results demonstrate that for PSK modulation the optimum choice of k and l is $k=1$ and $l=2$, and that the Gao SNR estimator with those parameters, $\hat{\rho}_{1,2}$, can outperform the $\mathrm{M_2M_4}$ estimator. The lookup table that would need to be used in employing the $\hat{\rho}_{1,2}$ estimator would simply consist in a number of samples of the function $f_{k,l}^{-1}$.

IV. Iterative Envelope-Based Estimator

In the following, we will derive an NDA envelope-based estimator using the first and second absolute moments with an iterative bias compensation procedure.

An intuitive envelope-based SNR estimator is simply the ratio of $E\{|y_n|\}^2$ to the variance $\mathrm{Var}\{|y_n|\}$. In fact, $\mathrm{E}\{|y_n|\}$ will tend to A for large ρ:

$$\lim_{\rho \longrightarrow \infty} \mathrm{E}\{|y_n|\} = A, \tag{18}$$

and the variance of the absolute value $\mathrm{Var}\{|y_n|\}$ will tend to N_0:

$$\lim_{\rho \longrightarrow \infty} \mathrm{Var}\{|y_n|\} = N_0. \tag{19}$$

For large SNR values, ρ is approximated by:

$$\hat{\rho} = \frac{\mathrm{E}\{|y_n|\}^2}{\mathrm{Var}\{|y_n|\}}. \tag{20}$$

This estimator is the same as the one proposed in [3] for BPSK modulation over coherent channels. We can derive an reduced bias version of this estimator for M-PSK modulation over complex channels as follows:

$$\hat{\rho}_0 = \frac{M_1^2}{2(M_2 - M_1^2)}. \tag{21}$$

On the other hand, this estimator still has a large bias (and therefore a large NMSE). Indeed, from (4) it is easy to verify that:

$$M_1 = \sqrt{S}\left(\Gamma(3/2)\frac{\exp(-\rho)}{\sqrt{\rho}}\mathrm{M}(3/2;1;\rho)\right). \tag{22}$$

In the estimator $\hat{\rho}_0$, M_1 used to estimate A is biased by the factor $\Gamma(3/2)\frac{\exp(-\rho)}{\sqrt{\rho}}\mathrm{M}(3/2;1;\rho)$. A fairly simple method to reduce this bias is to use the estimator $\hat{\rho}_0$ to evaluate the bias factor, then compensate for it. We calculate the estimates $\hat{A}$ and $\hat{\rho}$ by using the estimator expressed in (21). The bias is then reduced by dividing M_1 by $\Gamma(3/2)\frac{\exp(-\hat{\rho}_0)}{\sqrt{\hat{\rho}_0}}\mathrm{M}(3/2;1;\hat{\rho}_0)$, giving a new estimate $\hat{A}_1$:

$$\hat{A}_1 = \frac{\hat{A}}{\left(\Gamma(3/2)\frac{\exp(-\hat{\rho}_0)}{\sqrt{\hat{\rho}_0}}\mathrm{M}(3/2;1;\hat{\rho}_0)\right)}. \tag{23}$$

The new SNR estimator is then:

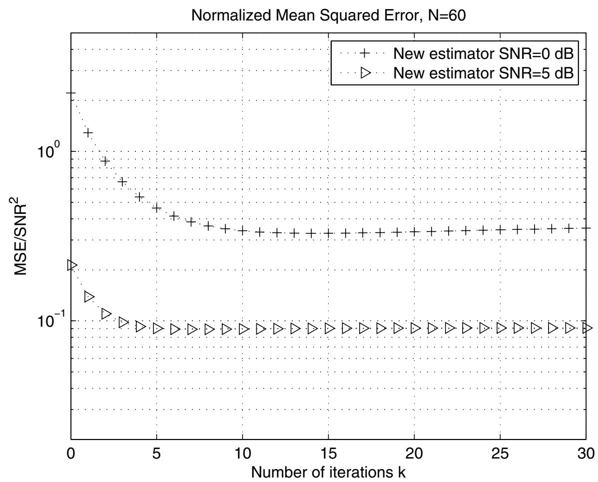

Fig. 2. True SNR normalized mean squared error of the new estimator with QPSK signals, $N=60$.

$$\hat{\rho}_1 = \frac{(\hat{A}_1)^2}{M_2 - (\hat{A}_1)^2}. \tag{24}$$

The procedure can be repeated to provide an iterative SNR estimate. Our iterative estimator is calculated using the following algorithm.

Input $y_1, ..., y_N$
Input I {number of iterations}

Initialization

calculate $\hat{A}_0 = \hat{M}_1 = \frac{1}{N}\sum_{n=1}^{n=N}|y_n|$

calculate $\hat{M}_2 = \frac{1}{N}\sum_{n=1}^{n=N}|y_n|^2$

calculate $\hat{\rho}_0 = \frac{\hat{A}_0^2}{2(\hat{M}_2 - \hat{A}_0^2)}$

For iteration $\mathbf{k = 1..I}$

$\hat{A}_k = \hat{A}_0 / \left(\Gamma(3/2)\frac{\exp(-\hat{\rho}_{k-1})}{\sqrt{\hat{\rho}_{k-1}}}\mathrm{M}(3/2;1;\hat{\rho}_{k-1})\right)$

$\hat{\rho}_k = \hat{A}_k^2 / (\hat{M}_2 - \hat{A}_k^2)$

End

Output $\hat{\rho}_I$

V. Simulation Results and Discussion

Simulation results are provided for the new SNR estimator. Specifically, QPSK and 8-PSK modulated signals over complex AWGN channels are simulated. For comparison, we provide also the performance of $\mathrm{M_2M_4}$, the AM and Gao

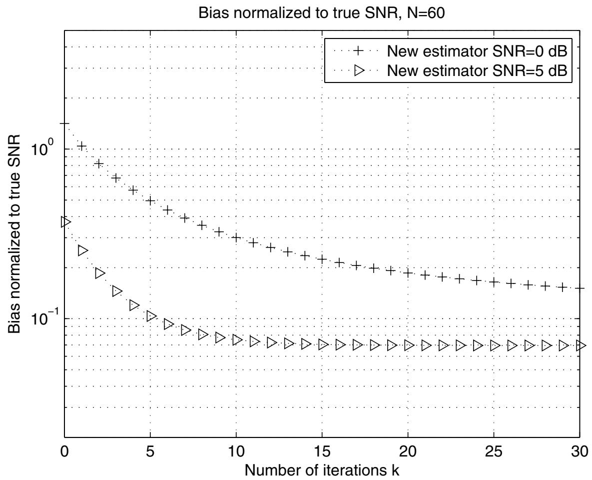

Fig. 3. Normalized bias of the new algorithm with QPSK signals, $N = 60$.

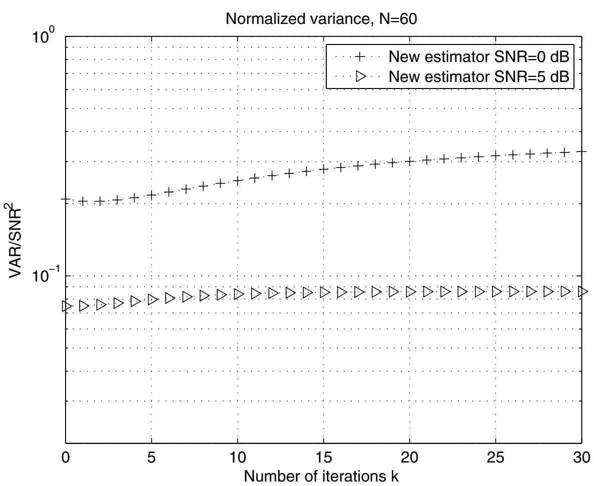

Fig. 4. Normalized variance of the new estimator with QPSK signals, $N = 60$.

estimators. For both modulations we choose 120 bits i.e N=60 and $N = 40$ symbols for QPSK and 8-PSK, respectively. For the AM estimator, a lookup table is needed. We used a grid with spacing of 0.001 corresponding to a table size 1000. For the Gao method, the size of the required table lookup size was taken very large (more than 8000 entries, with SNR ranging from lower than -76dB to 100dB in step of 0.02dB). We will also use the classical CRB [1] as a reference.

First of all, we start by looking at the performance of the new algorithm as a function of the number of iterations. For the sake of simplicity, we will only consider QPSK modulation. In Fig.2, we present the NMSE as a function of the number of iterations at different SNR values. It is immediately apparent from Fig. 2 that our algorithm can provide good performance with a relatively small number of iterations. A limited performance loss is observed as the number of iterations increases above $I = 15$. Notice that for each SNR value, we find an optimal number of iterations.

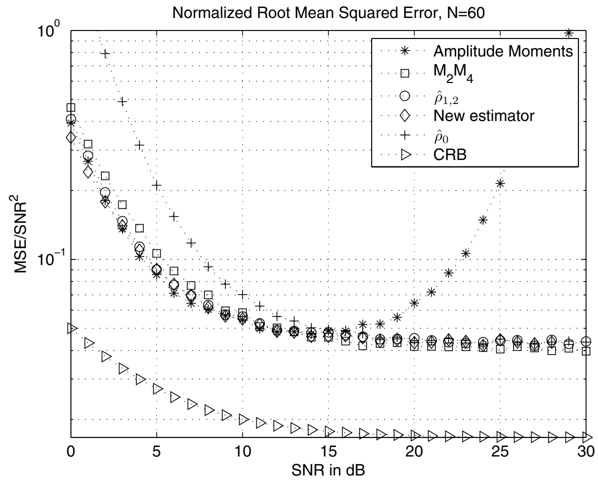

Fig. 5. True SNR normalized mean squared error of the estimators with QPSK signals, $N = 60$ and $I = 20$.

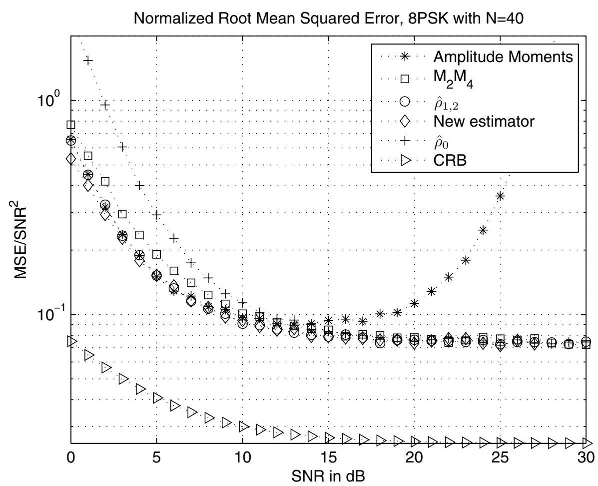

Fig. 6. True SNR normalized mean squared error of the estimators with 8-PSK signals, $N = 40$ and $I = 20$.

As we will see, there is a bias/variance tradeoff requiring an optimal number of iterations for each SNR value.

In Fig. 3, we plot the normalized bias as a function of the number of iterations. In terms of bias, it is observed that our algorithm converges in less than 30 iterations for the considered SNR values. As an example, at 0 dB the normalized bias is reduced by about 10.

In Fig. 4, we exhibit the normalized variance for different SNR values as a function of the number of iterations. We find that the new algorithm reduces the bias at the expense of an increase in variance. Hence, it yields to a bias-variance trade-off.

Figs. 5 and 6 show the NMSE as a function of SNR for the estimators with QPSK and 8-PSK signals over complex AWGN channels with data length of $N = 60$ and $N = 40$, respectively. The number of iterations is set to 20. As expected, the $\hat{\rho}_0$ estimator (initial condition of our recursive

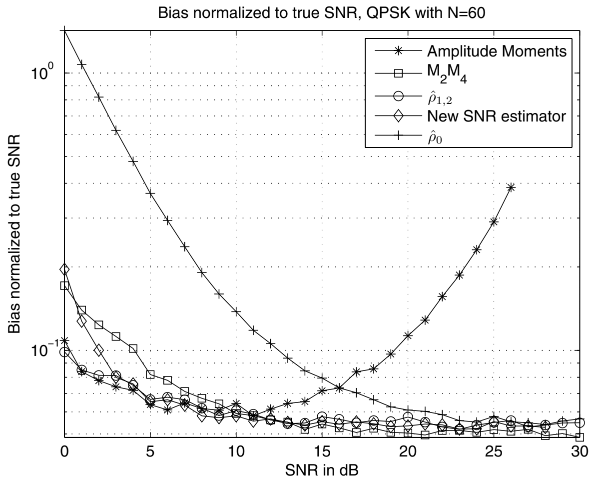

Fig. 7. Normalized bias of the estimators with QPSK signals, $N = 60$ and $I = 20$.

algorithm) performs poorly. The $\mathrm{M_2M_4}$ estimator is better than the estimator $\hat{\rho}_0$ but it is outperformed by the AM estimator at low SNR values. Notice here the degradation observed at high SNR for the AM. The NMSE of the newly proposed estimator is the smallest over the entire range of tested SNR values with only $I = 20$ iterations. The performance of the new estimator is similar to the Gao's performance without requiring a lookup table.

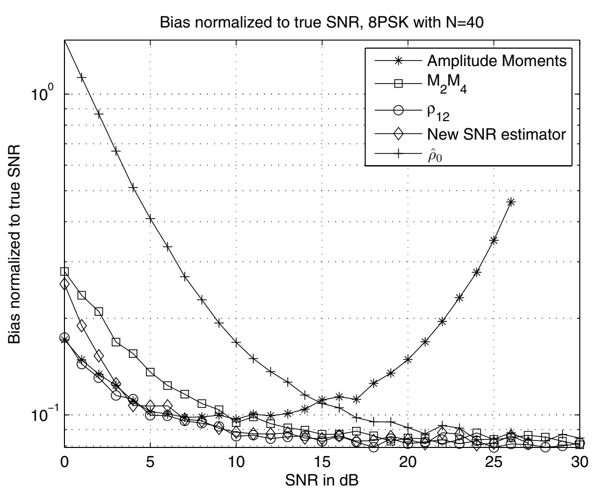

Fig. 8. Normalized bias of the estimators with 8-PSK signals, $N = 40$ and $I = 20$.

Figs. 7 and 8 compare the bias of the estimators under study. At low SNR, the bias of the AM estimator and Gao estimator is relatively small compared the one obtained with our algorithm. However, as seen in Figs. 5 and 6, the NMSE of the new estimator is still the smallest since the AM and Gao estimators exhibit a higher variance at low SNR. It can also be shown that with a higher number of iterations we can reduce the bias to reach the performances of the AM estimator. In contrast to our algorithm's performance at high SNR, the bias of the AM estimator increases dramatically for SNR $\geq$ 15dB. For AM and Gao estimators, the bias is determined by the lookup-table's size and the interpolation method, while for the new estimator is mainly determined by the number of iterations.

VI. Conclusion

In this paper we have derived a new SNR estimator for M-PSK signals. The algorithm is an estimator with iterative bias compensation. Simulations have shown that the new estimator exhibits performance gains over other previously proposed techniques for all values of SNR considered.

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