Impact of Angular Spread and DOA Estimation on the Performance of Wideband CDMA Array-Receivers[1]

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***Abstract*— Many research efforts were recently spent to address the challenging problem of DOA and angular spread estimation. However, to the best of our knowledge, no previous work has thoroughly investigated their specific impact on the performance of DOA-based antenna-array beamforming. In this contribution, we address this issue in the particular context of wideband CDMA using pilot-assisted or blind antenna-array receivers. In the process, we also assess whether the generalized channel-matched beamforming (*i.e.*, without DOA estimation or *a priori* knowledge of the spatial structure of the channel) offers a better alternative. Link-level simulation results in terms of required SNR at target BER of 1% suggest that wideband CDMA array-receivers, whether pilot-assisted or blind, are extremely sensitive to angular spread mismatches and that the benefits of exploiting the spatial structure of the channel (*i.e.*, estimation of DOA, angular spread, etc...) translate, at best, in negligible SNR gains when accurate channel identification is already implemented. These results call for implementing channel-matched instead of conventional DOA-based beamforming.**

I. Introduction

Motivated by the need for increased bandwidth efficiencies under limited availability of spectrum, smart antennas have been recently applied to improve the performance of future third generation (3G) wireless mobile communication networks and beyond (3G+). Indeed, with the capability and cost of signal processors improving, antenna arrays definitely allow now for the development and use of sophisticated and powerful array signal processing tools to boost the capacity, peak rates and coverage of wireless communications technologies.

One of the most popular antenna array processing techniques is DOA (direction of arrival)-based beamforming, which gained recently even stronger popularity with transmit beamforming. Conventional DOA-based beamforming consists in applying a spatial filter at the receiver or transmitter whose coefficients (*i.e.*, complex weights at the antennas) are tuned such that its radiation pattern points toward an *a priori*-estimated DOA of a plane wave along which the desired signal propagates to the receive or from the transmit antennas, respectively. In many typical wireless environments, however, sources are spatially distributed (*i.e.*, with an angular spread around a mean DOA) and no longer propagate along a plane wave due to local scattering in the surroundings of the mobile or base station. This mismatch between the nominal and real spatial structures of the channel has stirred tremendous research efforts to address the challenging problem posed upstream of estimating the DOA (or mean DOA) in the presence of an angular spread (cf. [1]- [5] to name a few). However, to the best of our knowledge, no previous work has thoroughly investigated the impact of angular spread and DOA estimation on the performance of DOA-based antenna-array beamforming.

In this contribution, we address this issue in the particular context of wideband CDMA using antenna-array receivers only[2]. In the process, we also assess whether the generalized channel-matched beamforming (*i.e.*, its coefficients match the channel gains estimated directly without DOA estimation or *a priori* knowledge of the spatial structure of the channel), is a better alternative to conventional DOA-based beamforming. In the following, we will simply refer to conventional DOA-based and generalized channel-matched beamformers as array-receivers with or without DOA estimation, respectively.

To carry out this study, we selected the spatio-temporal array-receiver (STAR) [6] as a common array-processing core structure. STAR, which has already been shown to outperform the 2D-RAKE array-receiver in synchronization, channel identification, signal combining and pilot exploitation [7], operates in both pilot-assisted and blind modes and does not exploit DOA estimation (*i.e*, implements channel-matched beamforming type). In this contribution, we endow it with an efficient and accurate DOA estimation technique [10] that attempts to improve channel estimation by structure fitting based on the *a priori* knowledge of the nominal spatial structure of the channel as a plane wave[3].

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[2]We will report in the near future on the results of an ongoing investigation dedicated to transmit beamforming.

[3]Angular spread estimation is not implemented. However, we will see from link-level evaluation results that its impact on performance can be qualitatively assessed reliably.

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II. Formulation and background

A. Data Model and Assumptions

We consider a single-user receiver structure on the uplink direction (portable-to-base station) of a cellular wideband CDMA system. Let us assume that each base station is equipped with M receiving antennas. We consider P propagation paths in a selective fading multipath environment. The user's binary phase shift keying bit sequence is first differentially encoded as $b_n = \underline{b}_n b_{n-1}$ at a rate $1/T_s$, where T_s is the bit duration. At the receiver, after despreading the data sequence, we form for each path $p = 1, \cdots, P$ the corresponding $M \times 1$ despread vector:

$$Z_{p,n} = G_{p,n}\varepsilon_{p,n}\psi_n b_n + N_{p,n} \quad (1)$$

where ψ_n^2 is the total received power and $\varepsilon_{p,n}^2$ is the normalized power fraction of the total power received over the p-th multipath (*i.e.*, $\sum_{p=1}^{P} \varepsilon_{p,n}^2 = 1$). The $M \times 1$ vector $G_{p,n}$, with norm $\sqrt{M}$, denotes the channel vector from the transmitter to the multi-antenna receiver over the $p-$th multipath. For more efficient joint space-time processing, the $M \times 1$ vectors $G_{p,n}$ are aligned, to generate the following $MP \times 1$ data observation vector:

$$\underline{Z}_n = [Z_{1,n}^T, \cdots, Z_{P,n}^T]^T = \underline{H}_n s_n + \underline{N}_n \ , \quad (2)$$

where $s_n = \psi_n b_n$ denotes the signal component, $\underline{H}_n = [\varepsilon_{1,n} G_{1,n}, \cdots, \varepsilon_{P,n} G_{P,n}]$ is the $MP \times 1$ vector with norm $\sqrt{M}$. $\underline{N}_n = [N_{1,n}^T, \cdots, N_{P,n}^T]^T$ is a space-time uncorralated Gaussian interference vector with mean zero and variance σ_N^2 after despreading of the data channel. The resulting input SNR after despreading is $SNR_{in} = \frac{\psi^2}{\sigma_N^2}$ per antenna element.

The vector channel $G_{p,n}$ is considered as a superposition of propagation path contributions associated to a continuum of angles of arrival (AOA) θ, propagation delays τ and Doppler angles ϕ as follows:

$$G_{p,n} = \iiint \mathrm{d}\theta \mathrm{d}\psi \alpha(\theta, \phi, \tau)\mathcal{F}(\theta) e^{j2\pi(nf_D T_s \cos(\phi) - f_c \tau)} \ , \quad (3)$$

where $\mathcal{F}(\theta)$ is the array propagation vector, defined as:

$$\mathcal{F}(\theta) = \left[1, e^{-2j\pi \sin(\theta)\frac{x_1}{\lambda}}, \cdots, e^{-2j\pi \sin(\theta)\frac{x_M}{\lambda}}\right]^T \ , \quad (4)$$

where λ is the wavelength, and x_m, $m = 1, \cdots, M$, are the sensor positions of a linear antenna array. f_D and f_c are the Doppler frequency, and the carrier frequency, respectively. $\alpha(\theta, \phi, \tau)$ is the path magnitude density function with respect to the DOA, Doppler angle and propagation delay having $f(\theta, \phi, \tau)$ as a joint power density function which is given by [8]:

$$f(\theta, \phi, \tau) = f_a(\tau) f_b(\phi) f_c(\theta) \ , \quad (5)$$

where $f_a(\tau)$, $f_b(\phi)$ and $f_c(\theta)$ are the power density functions with respect to the delay, the Doppler angle and the DOA for the p-th path, respectively. Note that the angular spread is defined in this paper as the width of $f_c(\theta)$ domain. In some papers, this parameter is defined as the standard deviation given by this distribution.

Note that for an angular spread $\Delta\theta = 0°$, the propagation follows a plane wave with a specified direction of arrival $\theta_{p,n}$. Therefore the channel vector $G_{p,n}$ in (3) becomes:

$$G_{p,n} = r_{p,n}\mathcal{F}(\theta_{p,n}) \ , \quad (6)$$

where $r_{p,n}$ is a phase shift due to Rayleigh fading. On the other hand, for an angular spread $\Delta\theta = 180°$, the power density function $f_c(\theta)$ represents a uniform distribution (*i.e.*, the channel coefficients are totally uncorrelated across antennas). Therefore, $G_{p,n}$ is a $M \times 1$ vector of normalized uncorrelated Rayleigh fading coefficients.

B. Blind STAR

At iteration n, the blind version of STAR uses the channel estimate $\hat{\underline{H}}_n$ to extract the data signal component by spatio-temporal MRC:

$$\hat{s}_n = \mathrm{Re}\left\{\frac{\hat{\underline{H}}_n^H \underline{Z}_n}{M}\right\} \ . \quad (7)$$

The DBPSK data sequence b_n is then estimated as $\hat{b}_n = \mathrm{sign}\{\hat{s}_n\}$. The receiver feeds back the estimate of the data signal component $\hat{s}_n$ (or $\hat{\psi}_n \hat{b}_n$) in a decision feedback identification (DFI) scheme to update the channel estimate as follows:

$$\hat{\underline{H}}_{n+1} = \hat{\underline{H}}_n + \mu(\underline{Z}_n - \hat{\underline{H}}_n \hat{s}_n)\hat{s}_n \ , \quad (8)$$

where μ is the adaptation step-size. This DFI scheme identifies the channel within a constant sign ambiguity $a = \pm 1$, thereby giving $\hat{\underline{H}}_n \simeq a\underline{H}_n$ and $\hat{b}_n \simeq ab_n$. However, differencial decoding of $\hat{b}_n$ resolves the sign ambiguity in the BPSK symbol estimates $\hat{\underline{b}}_n = \hat{b}_n \hat{b}_{n-1}$.

C. Pilot-Assisted STAR

The data model in sec. II-A is modified to achieve coherent transmission as follows: first we simply assign $b_n = \underline{b}_n$. Second we code-multiplex[4] the spread data with pilots. As in (1), at the receiver we despread the data channel and pilot to get both the observation vector $\underline{Z}_n^\delta$ and the pilot observation vector $\underline{Z}_n^\pi$. Note that $s_n^\pi = \xi\psi_n$ where s_n^π is the received pilot signal and ξ^2 denotes the allocated pilot-to-data power ratio. As in the blind version, pilot-assisted STAR applies the blind DFI in (8) to estimate the channel within a sign ambiguity a. It uses the extracted pilot symbols to resolve this sign ambiguity by averaging over consecutive blocks of A samples, since the pilot signal component $\hat{s}_n^\pi \simeq a\psi_n\xi$ carries a noisy value of a, hence, giving for $n = n' A, \cdots, (n' + 1)A - 1$:

$$\bar{s}_n^\pi = \sum_{i=1}^{A-1} \hat{s}_{n' A+i}^\pi / A \ , \quad (9)$$

[4]For lack of space, we present here only the pliot-channel version which shows approximatively the same performance as the pilot-symbol version where data and pilot are time-multiplexed [9].

$$\hat{a}_n = \text{sign}\{\bar{s}_n^{\pi}\}. \quad (10)$$

The BPSK symbol is finally estimated as $\hat{\underline{b}}_n = \hat{a}\hat{b}_n = \text{sign}\{\hat{a}\hat{s}_n^{\delta}\}$.

III. DOA Estimation in STAR

A new DOA-based estimation procedure [10] was developped to improve channel identification by fitting the structure of the channel estimate in its array manifold (*cf.* (6)) assuming plane-wave propagation (*i.e.*, $\Delta\theta = 0°$).

Denoting the channel estimate prior to this procedure as $[\tilde{H}_{1,n}^T, \cdots, \tilde{H}_{P,n}^T]^T$, we can write its p-th $M \times 1$ vector segment as:

$$\tilde{H}_{p,n} = a\varepsilon_{p,n} G_{p,n} + E_{p,n} = \epsilon_{p,n}\mathcal{F}(\theta_{p,n)}) + E_{p,n} \ , \quad (11)$$

where $\epsilon_{p,n} = a\varepsilon_{p,n} r_{p,n}$ and $E_{p,n}$ denotes the $M \times 1$ vector of identification errors over the p-th multipath. Providing means to extract both $\hat{\theta}_{p,n}$ and $\hat{\epsilon}_{p,n}$ from $\tilde{H}_{p,n}$ allows its recontruction as follows:

$$\hat{H}_{p,n} = \hat{\epsilon}_{p,n}\mathcal{F}(\hat{\theta}_{p,n}) \ , \quad (12)$$

yielding $\hat{\underline{H}}_n = [\hat{H}_{1,n}^T, \cdots, \hat{H}_{P,n}^T]^T$. This step is referred to as structure fitting. We implement it in a DOA tracking loop as follows.

First, we assume that structure fitting was already performed at iteration n, yielding $\hat{\underline{H}}_n$. Then we apply the DFI procedure of (8) to update the channel estimate $\tilde{\underline{H}}_n$ at iteration $n+1$. We extract $\hat{\epsilon}_{p,n+1}$ from $\tilde{H}_{p,n+1}$ by matched beamforming as follows:

$$\hat{\epsilon}_{p,n+1} = \frac{\mathcal{F}(\hat{\theta}_{p,n})^H \tilde{H}_{p,n+1}}{M} \ . \quad (13)$$

Finally, we update the multipath channel vector estimate by:

$$\tilde{\mathcal{F}}(\hat{\theta}_{p,n+1}) = \mathcal{F}(\hat{\theta}_{p,n}) + \mu_p \left(\tilde{H}_{p,n+1} - \mathcal{F}(\hat{\theta}_{p,n})\hat{\epsilon}_{p,n+1}\right)\hat{\epsilon}_{p,n+1}^* \ , \quad (14)$$

where μ_p is the adaptation step-size.

Assuming slow variations of $\theta_{p,n}$ compared to the symbol duration, estimation of $\hat{\kappa}_{p,n+1} = \frac{2\pi\sin(\hat{\theta}_{p,n+1})}{\lambda}$ from $\tilde{\mathcal{F}}(\hat{\theta}_{p,n+1})$ by simple update of $\hat{\kappa}_{p,n}$ is given by:

$$\hat{\kappa}_{p,n+1} = \hat{\kappa}_{p,n} - K^{-1}\left[\sum_{i=1}^{M} x_i \text{Im}(\Delta\mathcal{F}_i), \sum_{i=1}^{M} \text{Im}(\Delta\mathcal{F}_i)\right]^T \quad (15)$$

where $\Delta\mathcal{F}_i = \log(\tilde{\mathcal{F}}(\hat{\theta}_{p,n+1})e^{jx_i\hat{\kappa}_{p,n}})$ and K is defined as:

$$K = \begin{bmatrix} \sum_{i=1}^{M} x_i^2 & \sum_{i=1}^{M} x_i \\ \sum_{i=1}^{M} x_i & \sum_{i=1}^{M} 1 \end{bmatrix} . \quad (16)$$

Estimation of $\mathcal{F}(\hat{\theta}_{p,n+1})$ and $\hat{\underline{H}}_{n+1}$ in (12) using $\hat{\epsilon}_{p,n+1}$ in (13) and $\hat{\kappa}_{p,n+1}$ in (15) completes structure fitting at iteration $n+1$.

To achieve a more accurate DOA tracking, this procedure is disabled/reactivated if the path power is below or under a certain detection threshold noted as δ_{TH} (see [10] for details).

IV. Link-level performance evaluation

A. Simulation Setup

We consider a wideband CDMA system with 5 MHz bandwidth and $P = 3$ equal-power Rayleigh-fading paths propagating from directions with mean DOA values $\theta_1 = \pi/7$, $\theta_2 = -\pi/5$ and $\theta_3 = -\pi/3$ and an equal angular spread $\Delta\theta$. We use the vector channel model in [8] to generate the Rayleigh-fading channel coefficients from each spatially distributed multipath replica of the desired BPSK-modulated signal to the $M = 8$ or 16 antenna elements of a uniform linear array (ULA). Two representative mobile speeds of almost 5 and 50 Km/h, resulting in a Doppler shift of about $f_D = 9$ Hz and $f_D = 90$ Hz, respectively, at a carrier frequency of $f_c = 1.9$ GHz are examined. Power control (PC) requests an incremental change of ± 0.125 dB in transmitted power every 0.625 ms with a delay of 0.625 ms and an error of 10% over the PC bit command.

In the pilot-assisted version of STAR, the allocated pilot-to-data power ratio η^2 is set to 10%.

An analytical expression for the optimal step-size μ was previously derived in [7]. Its expression provides a minimum channel estimation misadjustement using the DFI scheme of (8). It was applied to both blind and reference-assisted versions of STAR. In this paper, the analytical expression for the optimal step-size μ is confirmed to be optimal for all values of angular spread $\Delta\theta$. Its expression is given by [7]:

$$\mu_{\text{opt}} = 2\left[\frac{\pi f_D T_s}{\sqrt{P}\bar{\psi}^2\sigma_N}\right]^{\frac{2}{3}} \ , \quad (17)$$

where $1/T_s = 19.2$ Kb/s and $\bar{\psi}^2 \simeq 1$.

B. Simulation Results

In Figs. 1 and 2, we present the link-level simulation results of STAR with and without DOA estimation, both in the pilot-assisted and blind modes, in terms of required SNR at a target BER of 1% (inferred from BER vs. SNR curves) versus the angular spread $\Delta\theta$ ranging from 0 (*i.e.*, plane wave and fully correlated) to π (*i.e.*, spatially diffused and totally uncorrelated). We also provide as a reference the performance curves of both pilot-assisted and blind STAR with perfect channel identification. Results suggest the following:

- At $\Delta\theta = 0$, STAR with or without DOA estimation requires about the same SNR level to achieve a BER of 1%, either in the pilot-assisted or blind mode. At less than half a dB from optimal performance in either case (see curves with perfect channel identification), STAR readily implements very accurate channel identification without DOA estimation. Hence, any further improvement in channel identification by additional DOA estimation and structure fitting translates in negligible link-level performance gains.
- As the angular spread increases, the performance of STAR without DOA estimation improves due to increased diversity, but saturates very quickly as the channel already

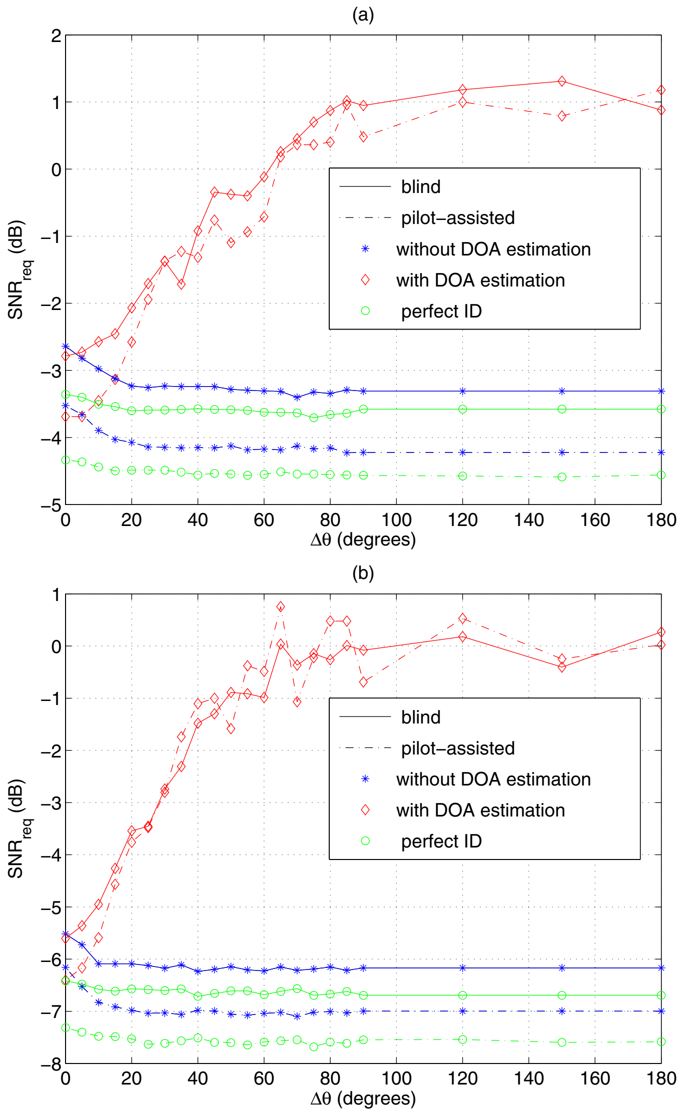

Fig. 1. Link-level simulation results of array receiver STAR at 5 Km/h with and without DOA estimation, both in the pilot-assisted and blind modes, in terms of required SNR at a target BER of 1% versus the angular spread $\Delta\theta$, using (a): $M = 8$ and (b): $M = 16$ ULA antenna elements.

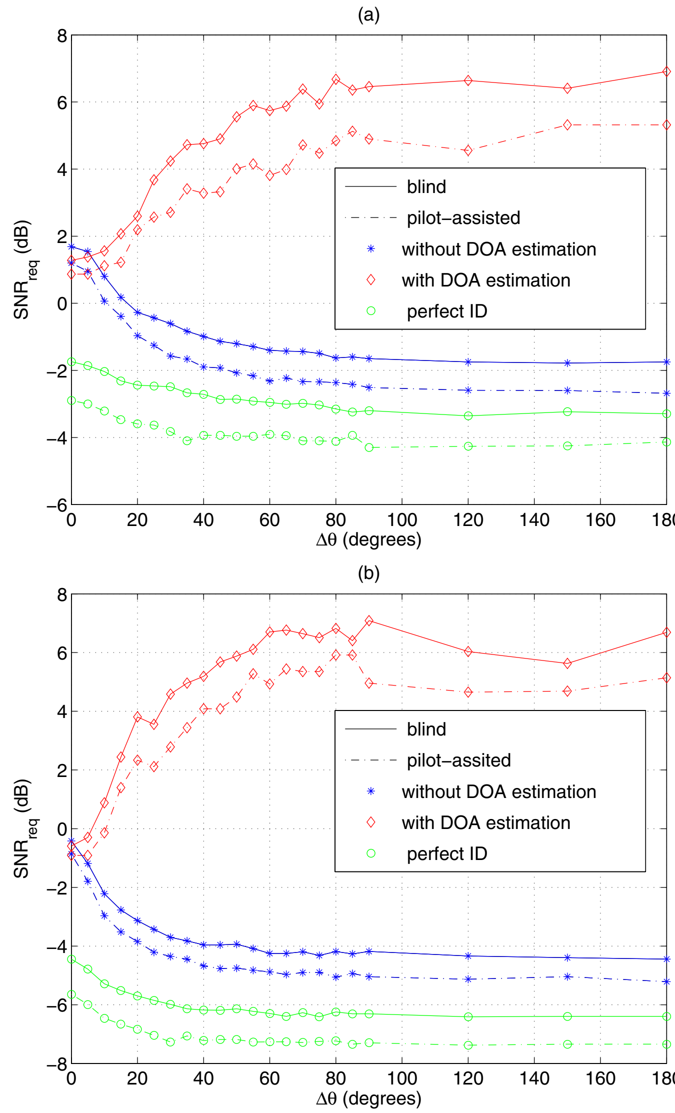

Fig. 2. Link-level simulation results of array receiver STAR at 50 Km/h with and without DOA estimation, both in the pilot-assisted and blind modes, in terms of required SNR at a target BER of 1% versus the angular spread $\Delta\theta$, using (a): $M = 8$ and (b): $M = 16$ ULA antenna elements.

exhibits a rich diversity situation with $P = 3$ paths and $M = 8$ or $M = 16$ antennas in addition to power control. A slower saturation is expected with fewer paths and/or antennas (notice that saturation is faster with $M = 16$ antennas).

• On the other hand, the performance of STAR with DOA estimation constantly degrades as the angular spread increases. This is hardly surprising since the mismatch between the nominal (assumed plane wave) and real spatial structures of the channel keeps widening. However, it is striking to note that performance losses are relatively more significant at smaller angular spreads, more so with a larger number of antennas (indeed the beamwidth, becoming narrower, captures less signal rays). At $\Delta\theta = 20$ degrees, SNR losses are of about 1 and 2 dB for both blind and pilot-assisted versions of STAR at 5 km/h for M=8 and 16 antennas, respectively. These losses deeply increase by an amount of about 2 dB at 50 km/h. Losses are such that blind STAR without DOA estimation performs better than pilot-assisted STAR with DOA estimation at 5 km/h for an angular spread larger than about 15.5 and 7.5 degrees (at intersection points) for M=8 and 16 antennas, respectively. At a higher speed, intersection points are much more closer to $\Delta\theta = 0$ (10 and 5 degrees for M=8 and 16 antennas, respectively).

• The question that arises immediately is whether additional angular spread estimation can help improve the link-level performance of STAR with DOA estimation.

Without recurring to implementation, the answer is no. First of all, estimation of the angular spread alone is not sufficient to reconstruct the channel. One needs to retrieve the channel coefficients of each ray from the angular spread. This is an extremely challenging task. Assume for a moment that a practical technique, if any, was developed to estimate such rays along with the angular spread to allow reliable reconstruction of the channel. Due to cumulative estimation errors, there is no reason to expect that accuracy of channel identification would be higher than that achieved in the absence of an angular spread where STAR with or without DOA estimation performed almost equally (cf. first bullet above). Therefore, at best, the benefits of exploiting the spatial structure of the channel in the presence of angular spreads will translate again in negligible link-level performance gains (if not losses) versus STAR without DOA estimation (which slightly improves its channel identification accuracy due to increasing diversity). Taking into account the computational cost incurred by the additional exploitation of the spatial structure of the channel versus its very limited gains in link-level performance, this study advocates the implementation of generalized channel-matched beamforming when using very accurate and efficient array-receivers such as STAR. DOA and angular spread estimation can be implemented in an open loop (*i.e.*, without exploitation in channel reconstruction) for localization or channel characterization purposes only.

V. Conclusions

In recent years, a great deal of effort was spent to address the challenging problem of DOA and angular spread estimation. However, no previous work has thoroughly investigated their specific impact on the performance of DOA-based antenna-array beamforming. In this contribution, we addressed this issue in the particular context of wideband CDMA using pilot-assisted or blind antenna-array receivers. In the process, we also assessed whether the generalized channel-matched beamforming offers a better alternative. Using an analytically optimized step-size in channel identification procedure (DFI), link-level simulation results in terms of required SNR at target BER of 1% suggest that wideband CDMA array-receivers, whether pilot-assisted or blind, are extremely sensitive to angular spread mismatches and that the benefits of exploiting the spatial structure of the channel translate, at best, in negligible SNR gains when accurate channel identification is already implemented.These results associated to the higher computational burden of the additional exploitation of the spatial structure call for implementing channel-matched instead of conventional DOA-based beamforming.

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