

A Downlink OFDM Switched-Beam Scheme with Joint Channel Estimation and Beam Selection Based on Kalman Filtering

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Abstract—This article addresses a new topic in OFDM with switched-beam systems, which is the exploitation of the OFDM symbols' structure to jointly perform the beam selection and channel estimation. Unlike previous works which were based on the symmetry of the channel to select the serving beam from the uplink of a MISO/MIMO system, our approach deals with the MISO downlink where the selection process is controlled at the mobile level. The method is based on the use of Kalman filtering with very spectrum-efficient overlapping pilots to separate the beams.

I. INTRODUCTION

With the high demand for broadband communications, in general, and the wireless communications, in particular, many research works have been engaged in the development of new revolutionary schemes of high capacity communication systems providing end-users with higher bit rates.

OFDM, which stands for Orthogonal Frequency Division Multiplexing, is one of the most promising technologies that has recently known a growing interest, due to its simplicity of implementation [1], and to the resistance it shows toward inter-symbol interference in severe multipath environments. This resistance comes from the fact that OFDM transforms high rate input symbols to low rate parallel transmitted symbols, therefore increasing the symbol duration and consequently reducing the ISI [2].

OFDM was selected to be used in the physical layer in many standards including : DAB [3], DVB [4], ADSL [5], Wireless LANs, especially HIPERLAN/2 [6] for ETSI and 802.11g for IEEE. Recently, it was also adopted for the physical layer of WiMAX [7].

Multiple antennas have proven their potential to increase significantly the capacity of wireless communication systems [8]. Particularly, when the spatial correlation is high between the channels seen by different antennas, beamforming becomes very attractive to achieve higher performance [9]. If we consider the small amount of feedback to adapt the antenna weights, and the ease of extension to the existing infrastructure at a minimum cost, switched-beam remains one of the most valuable techniques [10], [11].

Beam selection is one of the main issues encountered and dealt with in many recent works on switched-beam systems

[10], [12], [14]. [10] suggests that the received signal strength be simply used as a criterion to select the serving beam, and that the same selected beam be used for both the uplink and downlink, which is not always appropriate [15]. In CDMA-based networks, specific channels such as S-CPICH are used [12]-[13] for beam selection. As for switched-beam OFDM systems, [14] has presented a beam selection algorithm on the uplink based on two criteria: the signal strength and the peak-to-trough ratio (PTR), computed using the preamble defined in the 802.11a and 802.16 standards.

As mentioned earlier, most of the literature focuses on the uplink and the symmetry of the channel to assign beams to downlink users. Besides, in CDMA networks beam selection is performed easily thanks to the beam-dedicated orthogonal channels. But to the best of our knowledge, when dealing specifically with OFDM, the structure of OFDM symbols has never been taken into consideration in the process of beam selection.

In this work, we develop a new downlink switched-beam scheme for OFDM that is appropriate for situations where beam selection cannot rely solely on the uplink/downlink symmetry. This scheme, in addition, includes a new downlink pilot strategy which allows for the proper estimation of the beam channel and the selection of the most appropriate beam to be used by the base station. Furthermore, it implements very efficient beam selection jointly with channel estimation using Kalman filtering based on a special pilot overlapping structure.

The paper is organized as follows. In section 2, the system and the channel model are presented. The new method of joint beam selection and channel estimation is then developed in section 3. Simulation results are discussed in section 4. Conclusions are finally made in section 5.

II. SYSTEM MODEL

We consider the downlink where a mobile has a single isotropic receive antenna surrounded by uniformly distributed scatterers [16]. On the other hand, the base station, which is located high enough not to be shadowed by local scatterers (e.g. on a tower), has multiple transmit isotropic antennas. Furthermore, the base station and the mobile are supposed far

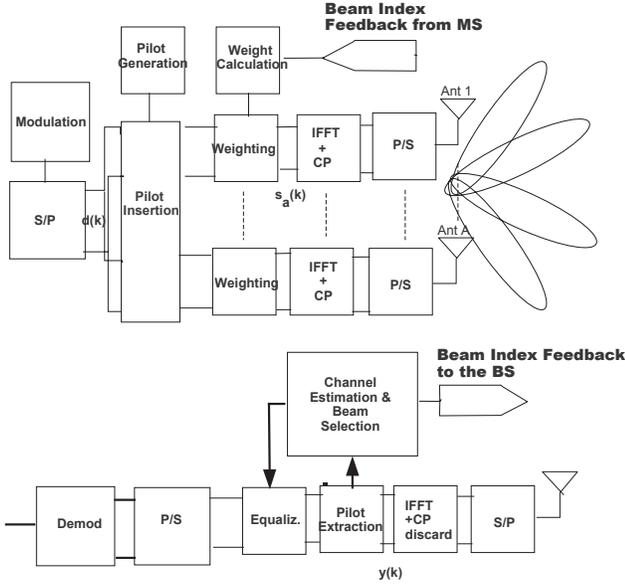


Fig. 1. OFDM MISO system with switched-beams

enough from one another as to create a near-planar wavefront over the antenna array surface.

We use an OFDM-based system with multiple antennas at the transmission and a single antenna at the reception as illustrated in figure 1. We assume that, there are A antennas at the transmission and that we have B users each associated to a separate beam in the sector. The base station transmits to each user in beam b , N parallel symbols denoted by $\mathbf{d}_b(k) = [d_{b,1}(k), \dots, d_{b,i}(k), \dots, d_{b,N}(k)]^T$, where k refers to the OFDM symbol index and the subscript i is the subcarrier index. Before processing this symbol by the inverse FFT (IFFT), it is duplicated as many times as the number of transmit antennas, then multiplied by the weight matrix used in each antenna, yielding in the a^{th} antenna branch the symbol $s_a(k)$:

$$\mathbf{s}_a(k) = \sum_{b=1}^B w_{a,b} \mathbf{d}_b(k), \quad (1)$$

where $w_{a,b}$ is the a^{th} antenna weight of the b^{th} beam. The weights are chosen to steer the transmitted data symbol towards a specific direction given by beam b . As can be noticed, all subcarriers are multiplied by the same scalar weight $w_{a,b}$ to steer them toward the b^{th} beam direction, since, for the sake of simplicity only and without loss of generalization, the relative frequency difference between subcarriers is assumed to remain very negligible compared to the main frequency carrier.

At any time interval when a signaling symbol should be sent, we can write:

$$\mathbf{s}_a = \sum_{b=1}^B w_{a,b} \mathbf{p}_b \quad (2)$$

where \mathbf{p}_b is the pilot allocated to the beam b . We consider a pilot structure similar to the one presented in [18] which uses phased pilot sequences, but adapted here to fit our downlink switched-beam system. The pilot for beam b can be written at the i^{th} subcarrier as:

$$p_{b,i} = \frac{1}{B} \exp(-j2\pi bi/B). \quad (3)$$

In the following, we use a notation similar to [17] with a generalization to the MISO systems case. After FFT processing at reception of the user in the β^{th} beam, the processed symbol $\mathbf{y}_\beta(k)$ can be written as:

$$\mathbf{y}_\beta(k) = \sum_{a=1}^A \mathbf{H}_{a,\beta}(k) \mathbf{s}_a(k) + \mathbf{F}_N \boldsymbol{\nu}_\beta(k), \quad (4)$$

where $\mathbf{H}_{a,\beta}(k)$ is a diagonal matrix of the transformed channel coefficients in the frequency domain between the a^{th} antenna and the user in the β^{th} beam, and \mathbf{F}_N is the Fourier matrix given below:

$$\mathbf{F}_N = \begin{bmatrix} 1 & 1 & \dots & \dots & 1 \\ 1 & \omega & \omega^2 & \dots & \omega^{N-1} \\ \vdots & \omega^2 & \omega^4 & \dots & \omega^{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{N-1} & \omega^{N-2} & \dots & \omega \end{bmatrix}, \quad (5)$$

where $\omega = e^{-j\frac{2\pi}{N}}$, and $\boldsymbol{\nu}_\beta(k)$ is the Gaussian white noise at time instant k .

Using (1) and (2), (4) can be reformulated as:

$$\mathbf{y}_\beta(k) = \sum_{b=1}^B \sum_{a=1}^A w_{a,b} \mathbf{H}_{a,\beta}(k) \mathbf{d}_b(k) + \mathbf{F}_N \boldsymbol{\nu}_\beta(k), \quad (6)$$

when k corresponds to a time when data symbol is transmitted, and as:

$$\mathbf{y}_\beta(k) = \sum_{b=1}^B \sum_{a=1}^A w_{a,b} \mathbf{H}_{a,\beta}(k) \mathbf{p}_b + \mathbf{F}_N \boldsymbol{\nu}_\beta(k), \quad (7)$$

when k is referring to a time corresponding to a pilot symbol.

Finally, by taking the diagonal matrix $\mathbf{H}_{b,\beta}(k) = \sum_{a=1}^A w_{a,b} \mathbf{H}_{a,\beta}(k)$, (6) and (7) are transformed to:

$$\mathbf{y}_\beta(k) = \sum_{b=1}^B \mathbf{H}_{b,\beta}(k) \mathbf{d}_b(k) + \mathbf{F}_N \boldsymbol{\nu}_\beta(k), \quad (8)$$

and

$$\mathbf{y}_\beta(k) = \sum_{b=1}^B \mathbf{H}_{b,\beta}(k) \mathbf{p}_b(k) + \mathbf{F}_N \boldsymbol{\nu}_\beta(k), \quad (9)$$

respectively.

III. JOINT BEAM SELECTION AND CHANNEL ESTIMATION

Joint Beam selection and channel estimation is performed using Kalman filtering. Our strategy for the downlink multi-beam scenario is based on the use of overlapping pilots to separate each beam. A similar approach has previously been adopted for multiuser signaling on the uplink of a single-antenna system [19].

First, we model the beam channel seen by one of the mobiles in subcarrier i as an AR(2) process as in [20]. Note that we omit the index β . Consider \mathbf{x}^s as the state variable which is of length 2. The state equation can be written as:

$$\mathbf{x}_i^s(k+1) = \mathbf{F}\mathbf{x}_i^s(k) + \mathbf{G}e_i^s(k), \quad (10)$$

where e_i^s is the innovation noise at subcarrier i , and

$$\mathbf{F} = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix}. \quad (11)$$

a_1 and a_2 are given in [20] as

$$a_1 = -2\rho \cos\left(\frac{\Omega_d}{\sqrt{2}}\right), \quad (12)$$

and

$$a_2 = \rho^2, \quad (13)$$

where f_d is the maximum Doppler frequency, and $\rho = 0.999 - 0.1\Omega_d$. Finally, $\Omega_d = 2\pi f_d T_p$ with T_p is the pilot period. The remaining array is given by:

$$\mathbf{G} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \quad (14)$$

The beam channel is related to the state variable by the following equation:

$$h_i^s(k) = \mathbf{C}\mathbf{x}_i^s(k), \quad (15)$$

where

$$\mathbf{C} = [1 \ 0]. \quad (16)$$

Generalization to all subcarriers is done by stacking $\mathbf{x}_i^s(k+1)$, $h_i^s(k)$ and $e_i^s(k)$ column-wise, yielding:

$$\mathbf{x}_b(k+1) = \text{diagb}(\mathbf{F})\mathbf{x}_b(k) + \text{diagb}(\mathbf{G})\mathbf{e}_b(k) \quad (17)$$

$$\mathbf{h}_b(k) = \text{diagb}(\mathbf{C})\mathbf{x}_b(k), \quad (18)$$

where $\text{diagb}(\mathbf{Y})$ is a block diagonal matrix made of the same matrix \mathbf{Y} .

The observation equation is given by:

$$\mathbf{y}_b(k) = \text{diag}(\mathbf{p}_b)\text{diagb}(\mathbf{C})\mathbf{x}_b(k) + \mathbf{v}_b(k), \quad (19)$$

where $\mathbf{v}_b(k)$ is the contribution of the white Gaussian noise through all the available subcarriers. $\text{diag}(\mathbf{u})$ is a diagonal matrix made of the array \mathbf{u} elements.

The system of equations is extended to the beam level by restacking the resulting arrays column-wise. The new equations are:

$$\mathbf{x}(k+1) = \text{diagb}(\text{diagb}(\mathbf{F}))\mathbf{x}(k) + \text{diagb}(\text{diagb}(\mathbf{G}))\mathbf{e}(k) \quad (20)$$

$$\mathbf{h}(k) = \text{diagb}(\text{diagb}(\mathbf{C}))\mathbf{x}(k), \quad (21)$$

and

$$\mathbf{y}(k) = [\text{diag}(\mathbf{p}_1), \dots, \text{diag}(\mathbf{p}_B)]\text{diagb}(\text{diagb}(\mathbf{C}))\mathbf{x}(k) + \mathbf{v}(k). \quad (22)$$

If we define:

$$\Phi = \text{diagb}(\text{diagb}(\mathbf{F})), \quad (23)$$

$$\epsilon(k) = \text{diagb}(\text{diagb}(\mathbf{G}))\mathbf{e}(k), \quad (24)$$

$$\Gamma = \text{diagb}(\text{diagb}(\mathbf{C})), \quad (25)$$

and

$$\Psi = [\text{diag}(\mathbf{p}_1), \dots, \text{diag}(\mathbf{p}_B)]\Gamma, \quad (26)$$

the notation of the state-space equations becomes:

$$\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \epsilon(k), \quad (27)$$

$$\mathbf{y}(k) = \Psi\mathbf{x}(k) + \mathbf{v}(k), \quad (28)$$

and

$$\mathbf{h}(k) = \Gamma\mathbf{x}(k). \quad (29)$$

The system is resolved using the Kalman algorithm [21] as follows:

Initialization

$$\hat{\mathbf{x}}(0|0) = \mathbf{0}$$

$$\mathbf{P}(0|0) = \delta\mathbf{I} \text{ with } \delta \text{ very small}$$

For iteration $k = 1..$

$$\hat{\mathbf{x}}(k|k-1) = \Phi\hat{\mathbf{x}}(k-1|k-1)$$

$$\mathbf{P}(k|k-1) = \Phi\mathbf{P}(k-1|k-1)\Phi^H + \mathbf{R}_\epsilon(k)$$

$$\mathbf{K}(k) = \mathbf{P}(k|k-1)\Psi^H[\Psi\mathbf{P}(k|k-1)\Psi^H + \mathbf{R}_v(k)]^{-1}$$

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)[\mathbf{y}(k) - \Psi\hat{\mathbf{x}}(k|k-1)]$$

$$\mathbf{P}(k|k) = [\mathbf{I} - \mathbf{K}(k)\Psi]\mathbf{P}(k|k-1),$$

where $\mathbf{R}_\epsilon(k)$ and $\mathbf{R}_v(k)$ are, respectively, the correlation matrices of the innovation noise ϵ and of the observation noise $\mathbf{v}(k)$. These statistics are assumed available at the mobile level.

The stacked channel estimate is given by:

$$\hat{\mathbf{h}}(k) = \Gamma\hat{\mathbf{x}}(k|k). \quad (30)$$

After de-stacking all the beam channel estimates, the mobile selects the best serving beam at time k , $\tilde{b}(k)$, by measuring the mean received power of each of them using the following criterion:

$$\tilde{b}(k) = \arg\max_{b \in [1 : B]} \frac{1}{N} \sum_{i=1}^N |\hat{h}_{b,i}(k)|^2, \quad (31)$$

then feeds the corresponding index $\tilde{b}(k)$ back to the base station in order to switch the beam appropriately in the coming transmissions.

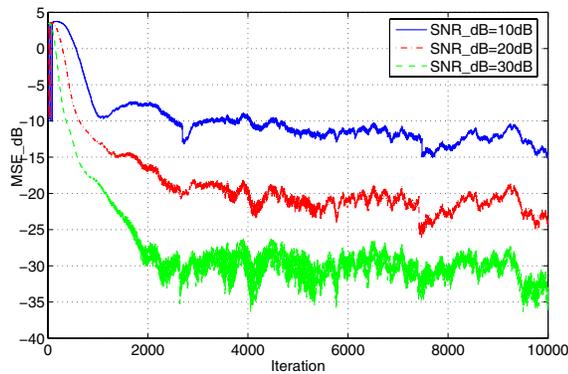


Fig. 2. Evolution of the MSE for different SNR_dB when the mobile has 3 kmph velocity.

IV. SIMULATIONS

We used the following parameters for the Monte-Carlo simulations:

- A 64 point FFT transform with an OFDM sample (QAM16 symbol) of duration $T_s = 0.2\mu s$, a cyclic prefix of length 1/4 of the total OFDM symbol.
- A total of $10000 \times 64 \times 4 = 2.560.000$ transmitted bits
- We consider the mono-user case.
- The system is operating at 5 GHz.
- The pilot rate is 10%.
- The channel used in simulations abides by the model of [22] with an angular spread $\sigma_\theta = 1.06^\circ$. It has 3 paths with an exponentially decaying power profile. The first path is supposed to arrive at time 0, while the second is at time $2T_s$ with $-3dB$ power compared to the first and the third is at time $6T_s$ with $-6dB$ compared to the first. The total transmitted power for a given user is normalized to 1. To simplify the interpretation of the results, we consider that the paths have identical Laplacian angular distributions with the same mean angle of arrival that sweeps the entire sector of $\frac{2\pi}{3}rad$ during the simulation time of $10000 \times (64 + 16) \times T_s$ (16 is the cyclic prefix length).
- 4 antennas are used. They are separated by half the wave length, and create a 4-beam pattern [23].

Let us analyze the performance of our system, firstly, in terms of channel estimation.

Figure 2 plots the evolution of the mean square error of the channel estimate using our proposed scheme, for different SNR values in dB when the mobile moves at low speed of 3 kmph. We can notice that the algorithm needs some time before it converges.

The performance of the proposed scheme in terms of beam selection is illustrated in figures 3 and 4, which show the evolution of the selected beam index with the iteration index of the simulation in a noisy and a "clean" environment,

¹The inner parameters of the channel model of [22] are set to $K = 10$, $\kappa = 0$, $\sigma_d = 2.65$ and $\Delta = 4^\circ$

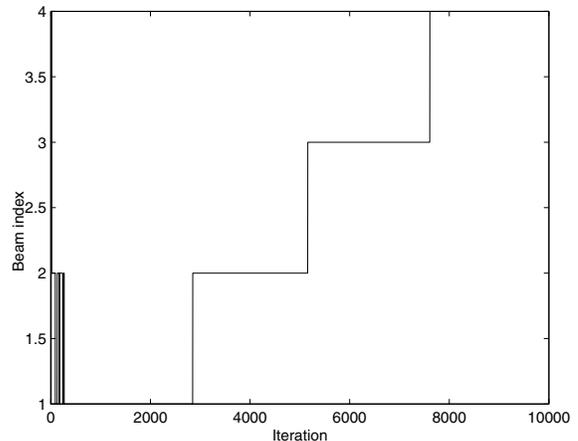


Fig. 3. Beam index selection vs iteration of the OFDM symbol in a noisy environment of 0 dB, when the mobile has 3 kmph velocity.

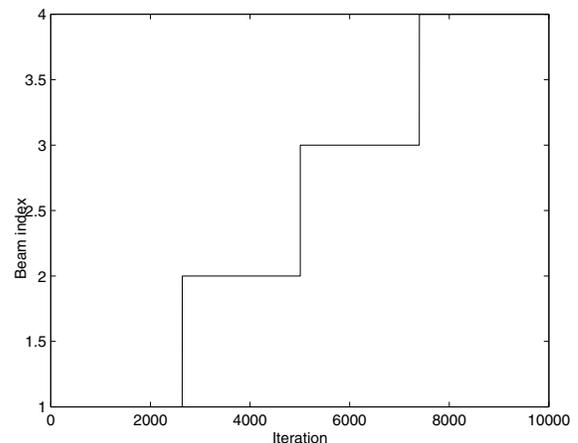


Fig. 4. Beam index selection vs iteration of the OFDM symbol in a clean environment of 30 dB, when the mobile has 3 kmph velocity.

respectively, when the mobile is sweeping the entire sector from beam 1 to beam 4 at a velocity of 3 kmph.

Results for a speed of 60 kmph are shown in figures 5 and 6. In our approach, the beam index is selected, actually, in accordance with the received power. The effect of noise can be clearly seen on the process of beam selection, through the previous figures.

Finally, the overall performance of the system using zero forcing (ZF) is shown in figures 7 and 8 which plot the bit error rate (BER) versus the SNR in dB for our proposed scheme for a speed of 3 kmph and 60 kmph, respectively. Note that the first fifth of the received symbols corresponding to the transient phase of the algorithm are not considered in the computation of the BER. In these figures, we compare the performance of our method with the performance of a smart box or a genie receiver (referred to as Genie in the figure), which exactly knows the channel, but uses the angular position of the mobile and not the best serving

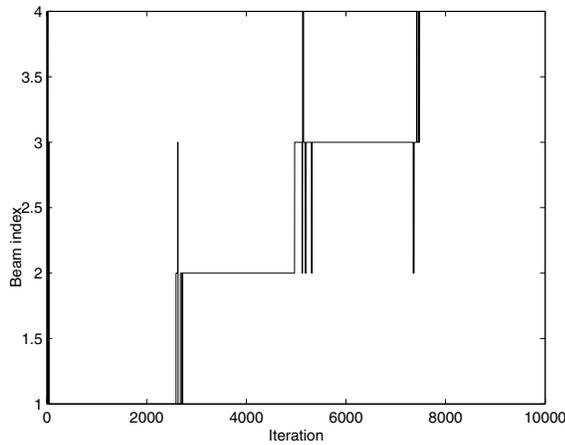


Fig. 5. Beam index selection vs iteration of the OFDM symbol in a noisy environment of 0 dB, when the mobile has 60 kmph velocity.

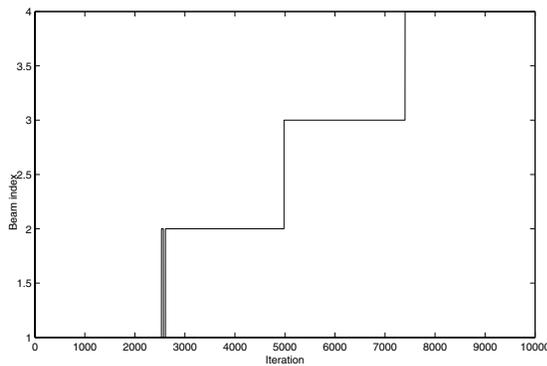


Fig. 6. Beam index selection vs iteration of the OFDM symbol in a clean environment of 30 dB, when the mobile has 60 kmph velocity.

beam in terms of received power to choose the serving beam. This could be the case if the BS controls the beam selection through the angular information extracted from the uplink. Besides, figures 7 and 8 include the performance of the approach based on subcarrier multiplexing [24] (referred to as SM in the figure). This method is based on the multiplexing of beam-related pilot symbols throughout the available subcarriers. At the reception, beam channels seen on each subcarrier are estimated, then the best serving beam is selected. For equalization purposes, an interpolation is used between estimated beam channels to fill the missing beam channel estimates in the subcarriers allocated to the nonselected beams.

We can notice that the proposed Kalman-based scheme is performing very close to the genie receiver for low speeds. But for higher speeds, the Kalman-based method becomes worse than the subcarrier multiplexing method. This is due to the mismatch of the used AR(2) model with the actual channel.

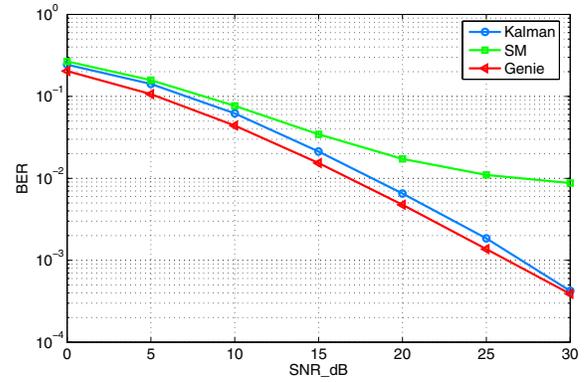


Fig. 7. BER vs SNR_dB for 3 kmph.

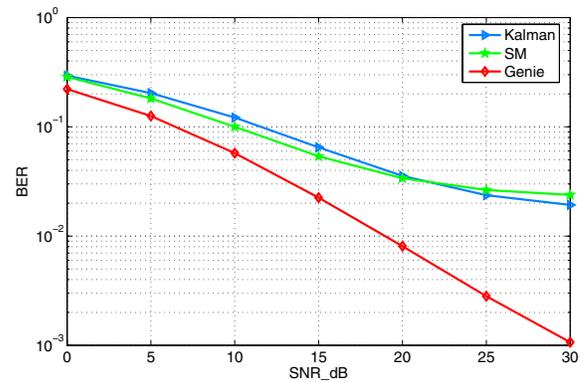


Fig. 8. BER vs SNR_dB for 60 kmph.

V. CONCLUSIONS

In this work, we have explored a new topic in OFDM switched-beam systems, by using the OFDM symbol structure to select the best serving beam in a MISO downlink link. The beam selection process is initiated by the mobile which feeds back the corresponding beam index to the base station. This approach makes our scheme always valid since no assumption of channel symmetry is required.

Our proposed scheme is based on Kalman filtering to jointly select the beam and estimate the channel. The algorithm assumes knowledge of some statistics about the channel, i.e., the innovation noise and white noise correlation matrices and the Doppler spread at the mobile. It uses spectrum-efficient overlapping pilots to estimate the channel seen in each beam separately, before the one with the highest mean received power over all the subcarriers is selected.

Results suggest that the beam selection algorithm and channel estimation perform very well at low speed, nearly as well as a genie receiver, and largely better the subcarrier multiplexing method [24]. For higher speeds, the performance of our method is subject to deterioration due to model mismatch between the used AR(2) model and the actual channel.

REFERENCES

- [1] S. B. Weinstein and P. M. Ebert, "Data transmission by frequency division multiplexing using the discrete Fourier transform," *IEEE Trans. Commun. Technol.*, vol. 19, pp. 628-634, October 1971.
- [2] L. J. Cimini Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. 33, no. 7, pp. 665-675, July 1985.
- [3] ETSI EN 300 401 V1.3.2, "Radio broadcasting systems; Digital audio broadcasting (DAB) to mobile, portable and fixed receivers," September 2000.
- [4] ETSI EN 301 701 V1.1.1, "Digital video broadcasting (DVB); OFDM modulation for microwave digital terrestrial television," August 2000.
- [5] ANSI T1.413 Issue 2, "Network and customer installation interfaces Asymmetric digital subscriber line (ADSL) metallic interface," November 1998.
- [6] ETSI TS 101 475 V1.2.2, "Broadband radio access networks (BRAN); HIPERLAN Type 2; Physical (PHY) layer," February 2001.
- [7] IEEE 802.16-2001, "IEEE standard for local and metropolitan area Nnetworks Part 16: air interface for fixed broadband wireless access systems," April 2002.
- [8] G. J. Foschini and M. J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311-335, Mars 1998.
- [9] K. I. Pedersen, P. E. Mogensen, and J. Ramiro-Moreno, "Application and performance of downlink beamforming techniques in UMTS," *IEEE Communications Magazine*, vol. 41, no. 10, pp. 134-143, October 2003.
- [10] M.-J. Ho, G.L. Stuber, and M.D. Austin, "Performance of switched-beam smart antennas for cellular radio systems," *IEEE Trans. Vehicular Technology*, vol. 47, no. 1, pp. 10-19, February 1998.
- [11] B. Allen, M. Beach, "On the analysis of switched beam antennas for the W-CDMA Downlink," *IEEE Trans. Vehicular Technology*, May 2004.
- [12] M. Schacht, E. Jugl, J. Mckenheim, and P. Jung, "A novel approach to consider spatial user information for scheduling in 3G mobile communication systems," *IEEE 5th International Conference on 3G Mobile Communication Technologies (3G2004)*, London, UK, 2004.
- [13] P. Monogioudis, K. Conner, D. Das, S. Gollamudi, J. A. C. Lee, A. L. Moustakas, S. Nagaraj, A. M. Rao, R. A. Soni, and Y. Yuan, "Intelligent antenna solutions for UMTS: Algorithms and simulation results," *IEEE Communications Magazine*, vol. 42, no. 10, October 2004.
- [14] L. Dong and M. A. Ingram, "Beam selection algorithm based on PTR metric and its synchronization performance," *IEEE Radio and Wireless Conference (RAWCON)*, Boston, MA, USA, 2003.
- [15] P. Zetterberg, *Mobile cellular communications with base station antenna arrays: Spectrum efficiency, algorithms and propagation models*, Ph.D. thesis, Royal Institute of Technology, Stockholm, Sweden, 1997.
- [16] W.C. Jakes Jr., *Microwave mobile communications*, John Wiley and Sons, New York, 1974.
- [17] M. Debbah, *Linear precoders for OFDM wireless communication*, Ph.D. thesis, ENS de Cachan, France, October 2002.
- [18] G. Auer, "Analysis of pilot-symbol aided channel estimation for OFDM systems with multiple transmit antennas," *IEEE ICC 2004*, Paris, France, June 2004.
- [19] M. Sternad and D. Aronsson, "Channel estimation and prediction for adaptive OFDMA/TDMA uplinks based on overlapping pilots," *IEEE ICASSP 2005*, Philadelphia, USA, March 2005.
- [20] L. Lindbom, A. Ahln, M. Sternad, and M. Falkenstrm, "Tracking of time-varying mobile radio channels, Part II: a case study," *IEEE Trans. Communications*, vol. 50, no.1, January 2002.
- [21] S. Haykin, *Adaptive filter theory*, 2nd edition, New Jersey, Prentice Hall, 1991.
- [22] Y. Semlani, A. Stéphenne, and S. Affes, "A new cost-effective MISO/SIMO channel model for macro-cellular system simulations", *Proc. of IEEE Canadian Workshop on Information Theory CWIT'05*, Montreal, Canada, to appear, June 6-8, 2005.
- [23] J. Butler and R. Lowe, "Beam forming matrix simplifies design of electronically scanned antennas," *Electron. Design*, vol. 9, pp. 170-173, April 1961.
- [24] Y. Semlani, A. Stéphenne, and S. Affes, "A downlink switched-beam OFDM pilot scheme based on subcarrier-multiplexing," *Technical Report*, no. 023-0705, INRS-EMT, Montreal, Canada, 2005.