# WIDEBAND MULTI-SOURCE BEAMFORMING WITH ADAPTIVE ARRAY LOCATION CALIBRATION AND DIRECTION FINDING

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## ABSTRACT

We present a novel adaptive algorithm in the frequency domain with a low order of arithmetic complexity for simultaneously performing beamforming, source direction finding and array shape calibration. The algorithm is proposed for multiple wideband sources, but could be applied to the narrowband case or to a single wideband source. The source signals are first estimated using a set of beamformers. These estimates are processed with the observation signals to track the steering vectors within the signal subspace. The adapted steering vectors are projected over the array manifold, to finally estimate the source directions and sensor positions. Simulations show the efficiency of the algorithm to achieve the proposed tasks.

## 1. INTRODUCTION

ADAPTIVE beamforming and localization algorithms can be extremely sensitive even to slight errors on the array shape, the sensors' gain and phase, and the DOAs (direction of arrival) [1].

To tackle this problem, beamforming algorithms such as [2] were developed to be robust to DOA errors. Their main task was not to correct the errors themselves, but to design beamformers with reasonably suboptimal performances in such a way that they become insensitive to them. Localization algorithms based on eigensubspace approaches and computationally more expensive were also proposed to directly find the DOAs. However, these methods remain sensitive to any modeling error related to the array shape or sensor characteristics.

Some algorithms were then proposed to reduce both the DOA errors and either sensor phase and gain [3,4] or location [5-7] uncertainties over the array sensors. These methods usually apply eigensubspace or ML approaches to determine a cost function of the vectors lying in it. An optimization of this function in the array manifold is usually made through a prohibitive search over the modeling parameters to find their optimal values [3,5-7]. However, these algorithms are either too costly to implement, or their applications are limited to certain forms of arrays.

In [8], we proposed an adaptive beamforming algorithm in the narrowband case, robust to localization errors and easy to implement. It is based on a tracking procedure of the steering vector, combined with a significant projection over the array manifold for location parameter extraction. The performance analysis and simulations proved the capacity of the algorithm to correct source location errors, and even to track mobile sources with efficient noise reduction and source extraction. We also generalized this algorithm to the multi-source [9] and wideband [10] cases, and proved its capability to track sources with source location errors within a practical range.

In this paper, we propose to reduce both DOA and sensor location errors in an adaptive manner through a significant extension of [8-10] to array location calibration with wideband signals. A generalization of the proposed algorithm to the calibration of phase and gain errors can be easily derived from the formulation made in this paper. The formulation is actually made but not limited to the case of a linear array, as shown in the generalization briefly described in section 4.

## 2. MATHEMATICAL FORMULATION

We consider the following model of $p$ wideband plane waves propagating signals received by a $m$-sensor linear array at time $t$ ($1 \le p \le m$):

$$X(t) = G(t) \otimes S(t) + N(t) , \qquad (1)$$

$$G(t) \triangleq [\delta(t - \tau_{i,j})] , \qquad (2)$$

where $X(t)$ is the $m$-dimensional observation vector, $S(t)$ is the desired $p \times 1$ column vector of wideband signals to be extracted, and $N(t)$ is an additive zero mean noise vector. $G(t)$ is the $m \times p$ matrix of Dirac impulse responses, and $\otimes$ denotes time convolution.

We consider here the case of far-field emitting sources, and do not take account of any possible phase or gain perturbations of the array sensors. $\tau_{i,j} \triangleq \frac{\xi_i \sin(\phi_j)}{C}$ is the propagation time delay from the $j^{\text{th}}$ source to the $i^{\text{th}}$ array sensor. $C$ is the celerity, $\Xi = [\xi_1, \xi_2, \cdots, \xi_m]^T$ is the vector of array sensor locations, and $\Phi = [\phi_1, \phi_2, \cdots, \phi_p]^T$ is the vector of source DOAs.

We assume that the number $p$ and the DOAs of all the point sources are approximately initialized by a localization technique or given by the vector $\hat{\Phi}_0 = [\hat{\phi}_{1,0}, \cdots, \hat{\phi}_{p,0}]^T$. Those sources which are not localized will have relatively small power, and will be confused with spatially diffuse

noise. Moreover, we assume that the $m$ array sensor positons are given with initial errors in a reasonably limited range by the vector $\hat{\Xi}_0 = [\hat{\xi}_{1,0}, \hat{\xi}_{2,0}, \cdots, \hat{\xi}_{m,0}]^T$. As well, $S$ and $N$ are assumed to be mutually independent.
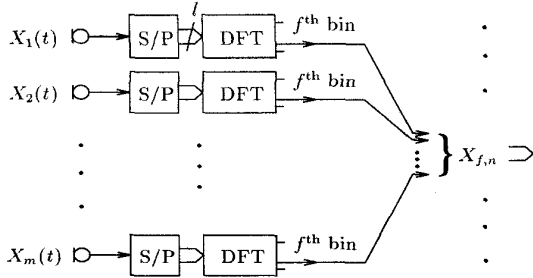


Figure 1: Serial to parallel and transform to the frequency domain of the observation signals.

Taking now the Discrete Fourier Transform (DFT) of the observation signals over $l$-sample blocks[1] as in [10], we can transpose the problem to the narrowband case as follows (see Figure 1):

$$X_{f,n} = G_{f,n}S_{f,n} + N_{f,n} , \qquad (3)$$

where the subscripts $f = 0, \frac{1}{l}, \cdots, \frac{l-1}{l}$ and $n$ denote the DFT of the indexed quantity respectively at the normalized frequency $f$ and the block iteration $n$. The DFT of $G(t)$, $G_f$, can be written as follows:

$$G_f = [F(\theta_1, \Xi, f), \cdots, F(\theta_p, \Xi, f)] , \qquad (4)$$
$$F(\theta, \Xi, f) = [e^{-jf\xi_1\theta}, e^{-jf\xi_2\theta}, \cdots, e^{-jf\xi_m\theta}]^T .$$

$F$ is a modeling function of a plane waves propagation of a narrowband source around a carrier frequency $f$, where $\theta \triangleq \frac{2\pi\sin(\phi)}{C}$ is the wavenumber at the normalized sampling frequency.

Given the propagation model $F(\theta, \Xi, f)$, the initial steering vectors $\hat{G}_{f,0} = [F(\hat{\theta}_{1,0}, \hat{\Xi}_0, f), \cdots, F(\hat{\theta}_{p,0}, \hat{\Xi}_0, f)]$ must be simultaneously and accurately time-adapted and corrected so as to perform calibration from the unique observations available $X_{f,n}$. We propose in the following an algorithm for adaptively correcting the source wavenumbers $\Theta_n$ (i.e. the DOAs $\hat{\Phi}_n$), and simultaneously calibrating the array sensor locations $\hat{\Xi}_n$.

## 3. PROPOSED ALGORITHM

Given the observation vector $X_{f,n}$, the source signals vector $S_{f,n} = [s_{1,f,n}, \cdots, s_{p,f,n}]^T$ is first estimated using a set of $p$ classical beamformers $H_{f,n}$ for $f = 0, \cdots, \frac{l-1}{l}$ as follows (see Figure 2):

$$\hat{S}_{f,n} = H_{f,n}^H X_{f,n} . \qquad (5)$$

We actually propose the following $m \times p$ conventional beamforming matrix:

$$H_{f,n} = \hat{G}_{f,n}(\hat{G}_{f,n}^H \hat{G}_{f,n})^{-1} , \qquad (6)$$

[1] $l$ is assumed to be large in comparison to the wavefront transit time across the array sensors.

which satisfies the equation $H_{f,n}^H \hat{G}_{f,n} = I_p$, where $I_p$ is a $p \times p$ identity matrix. In [9], $H_{f,n}$ is shown to have optimal performance in the presence of uncorrelated white noise. If the number of sensors $m$ is relatively large, we can avoid the matrix inversion in (6). Weighting functions $\mathcal{W}$ such as Kaiser or Hanning windows of length $m$ could be used instead at the array sensors to minimize the sidelobes particularly at the jamming locations, by $H_{f,n} = \text{diag}(\mathcal{W}) \, \hat{G}_{f,n}$. Parallel GSC (generalized sidelobe canceller) beamformers could be used instead, if the $p$ sources are in addition mutually uncorrelated [8-9].

The estimated frequency components $\hat{S}_{f,n}$ allow the synthesis of the desired wideband signals in the time domain using the OLS (overlap save) technique.
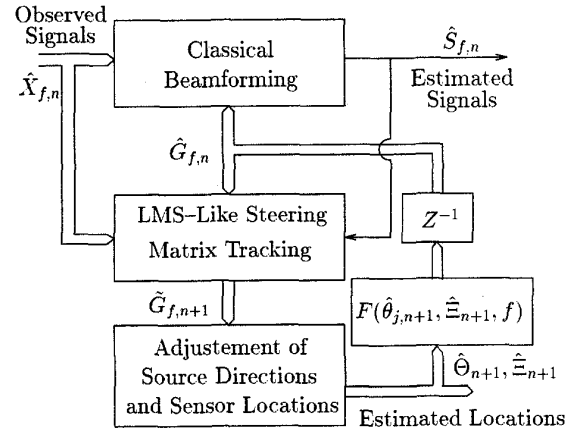


Figure 2: The block diagram at frequency $f$.

In [8-9], we proposed signal subspace tracking procedures in the narrowband case, which are able to track the steering vectors in the array manifold from the observed and estimated signals $X_{f,n}$ and $\hat{S}_{f,n}$. In the wideband case, we applied the tracking equations for a single desired source at a given frequency bin, and adapted the analysis results to it [10]. We proved a combined tracking in a well selected set of say $q$ "tracking frequencies" $f_1, f_2, \cdots, f_q$ to perform better to some extent [10].

For $f = f_1, f_2, \cdots, f_q$, we thus correct the steering vectors $G_f$ in the same way as in [9] at each block iteration as shown in Figure 2:

$$\tilde{G}_{f,n+1} = \hat{G}_{f,n} + \mu \, (X_{f,n} - \hat{G}_{f,n}\hat{S}_{f,n}) \, \hat{S}_{f,n}^H , \qquad (7)$$

where $\mu$ is a given step-size possibly including a normalization factor. Of course, it is not readily defined that the updated steering vectors $\tilde{G}_{f,n}$ belong to the array manifold. Hence, we should project them on it so as to extract both the DOAs and the sensor positions and to reconstruct $\hat{G}_{f,n}$. This can be achieved by using a linearization technique based on a first order Taylor series expansion of the propagation modeling function $F$ in (4). We have already explained this procedure when applied to DOA extraction alone. We have interpreted it as a linear regression of the adapted steering vector phase components over the $m$ sensor positions [8] (see also a similar approach in [4]). We

equivalently extract the source wavenumbers as follows (see Figure 2):

$$\hat{\Theta}_{f,n+1} = \hat{\Theta}_{f,n} - \frac{\arg_\Delta(\tilde{G}_{f,n+1}, \hat{G}_{f,n})^T \hat{\Xi}_n}{f \hat{\Xi}_n^T \hat{\Xi}_n}, \qquad (8)$$

where the array center is assumed to be at the origin (*i.e.* $1_m^T \hat{\Xi}_n = 0$ where $1_m = [1, 1, \cdots, 1]^T$), and $\arg_\Delta(.)$ is defined for any couple of $m \times p$ matrices $A = [a_{i,j}]$ and $B = [b_{i,j}]$ such that $\arg_\Delta(A, B) = [\arg(a_{i,j} \, b_{i,j}^H)]$. We already proved in [10] that the combination of the $q$ wavenumber estimates yields a lower misadjustment and speeds up the convergence when compared to the narrowband case (*i.e.* estimation in one frequency). If we assume for simplicity the signal and also the noise powers to be quite equal at all tracking frequencies, then we have (see [10] for more details):

$$\hat{\Theta}_{n+1} \triangleq \frac{\sum_{k=1}^q f_k^2 \hat{\Theta}_{f_k, n+1}}{\sum_{k=1}^q f_k^2}. \qquad (9)$$

We now introduce an additional step allowing the extraction of the sensor positions following each DOA adjustment. By analogy to equations (8) and (9), we perform another linear regression of the steering vector phase components over the $p$ source wavenumbers to estimate the locations of the sensors:

$$\hat{\Xi}_{f,n+1} = \Pi_m \left( \hat{\Xi}_{f,n} - \frac{\arg_\Delta(\tilde{G}_{f,n+1}, \hat{G}_{f,n}) \hat{\Theta}_{n+1}}{f \hat{\Theta}_{n+1}^T \hat{\Theta}_{n+1}} \right). \qquad (10)$$

where $\Pi_m \triangleq I_m - \frac{1_m 1_m^T}{m}$ is the projector orthogonal to the $m \times 1$ column vector $1_m$. In the same manner, we also combine the $q$ estimates of $\Xi$ as follows:

$$\hat{\Xi}_{n+1} \triangleq \frac{\sum_{k=1}^q f_k^2 \hat{\Xi}_{f_k, n+1}}{\sum_{k=1}^q f_k^2}. \qquad (11)$$

The improvement in performance due to combination of array sensor calibration at different frequencies is also confirmed by simulations.

In general, combinations in (9) and (11) can be seen as weighted regressions over $p \times q$ elements. Particularly in the presence of a single source (*i.e.* $p = 1$), (11) performs a simple regression over the wavenumbers at different frequencies on condition that $\theta \neq 0$.

Finally, we can see that calibration is possible either in the presence of well located multiple sources possibly narrowband, or a single wideband source, as we can benefit from $p \times q$ observations for each sensor ($1 \leq q \leq l$).

## 4. SIMULATION RESULTS

For simulations, we consider $p = 5$ uncorrelated wideband sources with quite equal powers, corrupted by an uncorrelated white noise at a mean SNR (signal to noise ratio) of 10 dB. We select $q = 3$ tracking frequencies for combination. We simultaneously run the algorithm with and without calibration, using or not equations (10) and (11).

In Figure 3-a, we first test a linear array of $m = 16$ sensors supposed to be equidistant at half the wavelength. The

calibration step enables the algorithm to reduce the MSE (mean square error) of sensor location by approximately 20 dB within 1000 iterations, as shown in Figure 3-b. Both running versions significantly reduce the MSE of DOA localization to the range of $10^{-1}$ deg$^2$ as shown in Figure 3-c, although calibration slightly improves the results. We actually avoid any ambiguity arising from a translation of the array as we constraint its center to be at the origin. This corresponds to a signal time delay estimation whose effect is not of the scope of this paper. We hence take the true array center at the origin. We see however the ambiguity arising from the multiplicative factor $\alpha$ between $\Xi$ and $\Theta$ as $\frac{\Xi}{\alpha} \times \alpha\Theta = \Xi\Theta$, although $\alpha$ is typically very close to 1 at convergence. Despite this negligible effect, the steering vectors are well identified with calibration. The MSE of signal estimation is reduced by 12 dB ($10 \log_{10}(m)$), the best gain achievable by the beamforming matrix in (6) in the presence of uncorrelated white noise [9].
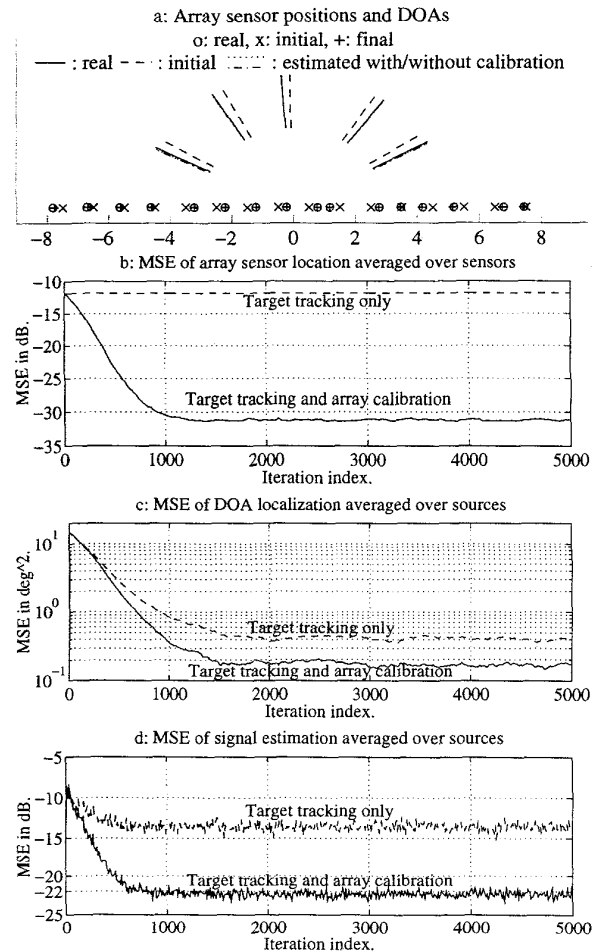


Figure 3: Performances in sensor and DOA localization and source extraction for a linear array.

Other simulations not shown in this paper, made with $p = 1$ and $q = 5$, prove the algorithm to have comparable performance.

**a: Array sensor positions and DOAs**

—— : real  - - - : initial  ⋯⋯ : estimated with/without calibration
o: real, x: initial, +: final

**b: MSE of array sensor location averaged over sensors**

Target tracking only

Target tracking and array calibration

**c: MSE of DOA localization averaged over sources**

Target tracking only

Target tracking and array calibration

**d: MSE of signal estimation averaged over sources**

Target tracking only
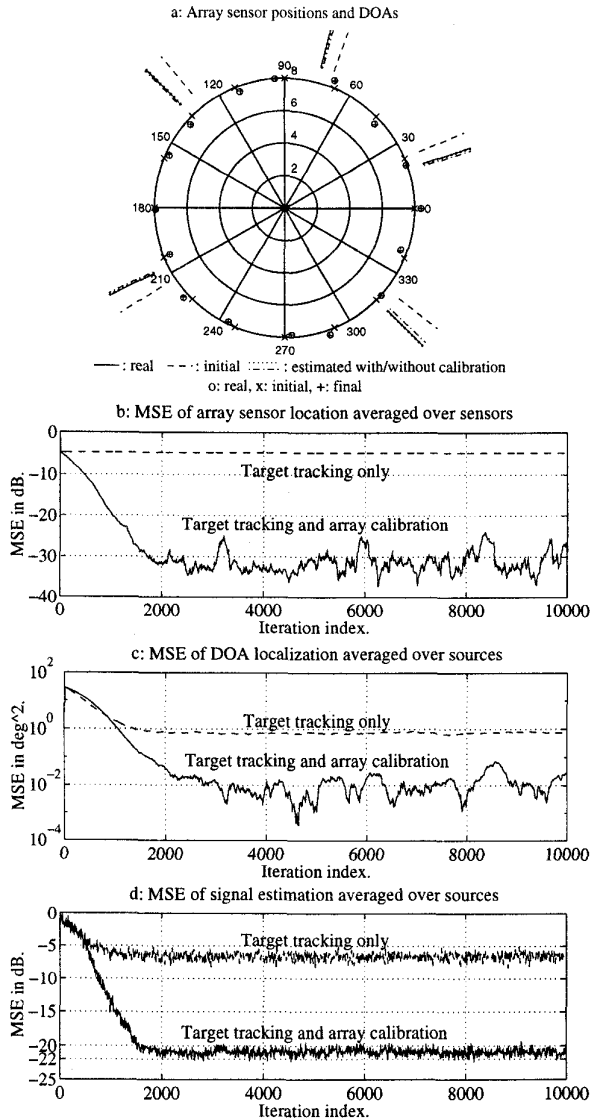
Target tracking and array calibration

Figure 4: Performances in sensor and DOA localization and source extraction for a circular array.

We secondly generalize the algorithm to a 2 dimensional array. To do so, we redefine $\xi_i = [\xi_i^x, \xi_i^y]^T$ and $\theta_j = [\theta_j^x, \theta_j^y]^T$ where $\theta_j^x = \frac{2\pi \sin(\phi_j)}{C}$ and $\theta_j^y = \frac{2\pi \cos(\phi_j)}{C}$. We replace (8) by a pair of equations where $\Theta$ and $\Xi$ are respectively replaced by $\Theta^x$ and $\Xi^x$, $\Theta^y$ and $\Xi^y$. In the same manner, we easily adapt equations (9) to (11) although we constaint the wavenumber $\theta_j$ to have constant norm $\|\theta_j\| = \frac{2\pi}{C}$. For illustration, we consider a circular array as shown in Figure 4-a. Figure 4-b shows the algorithm to handle stronger sensor location errors. In Figure 4-c, the MSE of DOA localization is reduced to the range of $10^{-3}$ deg$^2$. Initial errors are so high that the MSE in signal estimation is 0 dB at $t = 0$. The algorithm is able to reach the optimal performance at convergence after 2000 iterations as shown

in Figure 4-d. The additional ambiguity due to rotations is not assessed in this paper.

## 5. CONCLUSION

We proposed in this paper a low order of complexity algorithm for wideband multi-source beamforming, adaptive array location calibration and DOA localization. We proved by simulations its efficiency to achieve the proposed tasks in the case of linear or 2 dimensional arrays, although it can be tested in a 3 dimensional space. The performance improve to some extent with increasing numbers of emitting sources and tracking frequencies. Particularly with low tracking frequencies, the algorithm is able to reduce higher initial location errors. The combination with higher tracking frequencies speeds up convergence and reduces the misadjustments.

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